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ILLINIUM—THE NEW RARE EARTH.*

BY

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WORK on the rare earth group of elements was begun at the University of Illinois about 1909 under the inspiring leadership of Dr. Clarence W. Balke. During the early years much attention was given to the methods of separating the members of this group from one another, as well as from other chemical elements. Particular attention was directed to a study of that portion of the rare earth group which is usually designated as the yttrium sub-group. The material used came from a variety of sources, by far the most important being a liberal quantity of monazite residues which were generously donated by the Welsbach Mantle Company through its chief chemist, Dr. Harlan S. Miner.

Three distinct contributions resulted from this work in its early years. (1) The refinement of existing methods of separation made possible more rapid and more complete isolation of many of the members of the rare earth group. (2) Many atomic weights were determined upon materials which had been purified with extreme care. (3) The accumulation of considerable quantities of relatively pure compounds of certain rare earth elements made it possible to engage in various lines of research based upon these elements and to coöperate in many important investigations in other laboratories.

* Presented at the Stated Meeting held Wednesday, February 16, 1927.

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One of the problems for which material was furnished in a coöperative plan was in progress at the United States Bureau of Standards. An attempt was being made to map the red and infra-red region of the spectra of all the known elements. In the pursuit of this problem difficulty had been encountered in obtaining samples of the rare earth group which were of requisite purity. As a result a plan was agreed upon by which the University of Illinois furnished some of its materials both of the rare earth group and of other rare elements while the Bureau of Standards made precision measurements of the lines found in the arc spectra of these substances. The results have been published [1] in the Scientific Papers of the Bureau.

In publishing the work on neodymium and samarium, Dr. C. C. Kiess defines [2] about 1500 lines for each of these elements. In addition there is given the wave-length of 130 faint lines which are common to both spectra. He says: "These lines are of unknown origin and may belong to the missing element of order number 61, coming between Nd and Sa . . ." "When the tabulated wave-lengths for neodymium and samarium were intercompared, it was found that many lines, for the most part faint, were common to the spectra of both. It has not been possible to identify them with the spectra of known elements, and, therefore, they have been brought together in a separate table. While it would be a hasty conclusion to regard these lines as characteristic of a new element, yet the idea seems plausible in view of the fact that the present accepted scheme of the periodic table of elements requires a new element of atomic number 61 coming between neodymium and samarium.[3] Eder [4] has published similar lists of unidentified lines which he suggests may belong to elements not yet isolated, but none of the lines of our table agree with those in his lists."

Following this suggestion, work was begun upon the concentration and identification of the substance which produced the unknown lines. Dr. L. F. Yntema, working as Fellow of the National Research Council, carried out X-ray examination in the Sloane Laboratory of Physics at Yale, while fractionations were being continued in Urbana. The result of two years' work was the addition of five arc-spectrum lines toward the violet and the conclusion [5] that if number 61 were present at all, it must be in quantities of less than one part in a thousand.

It was evident that some method of obtaining a greater concentration must be found before the identity of the element could be established. Accordingly a study was made of the salts available for use in the fractional crystallization of this part of the group.

Previous attempts at concentration had been made by the fractional crystallization of the double magnesium nitrate. In

TABLE I.

Lines Possible in This Region Other Than Element 61.

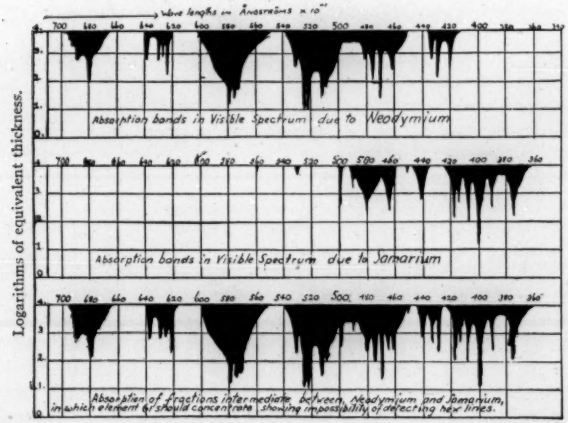
Wave-length.	Element.	Series.		Wave-length.	Element.	Series.
2.270	La	$L\beta_7$				
2.273	Cr	$K\alpha_2$				
2.277	Ce	$L\beta_6$	Lines of	2.067	Cr	$K\beta_2$
2.277	La	$L\beta_9$	first order	2.072	Ba	$L\gamma_4$
2.280	Va	$K\beta_1$	in	2.080	Cr	$K\beta_1$
2.285	Cr	$K\alpha_1$	this region	2.082	Ho	$L\rho$
2.285	La	$L\beta_{10}$				
2.285	Va	$K\beta^1$				
				1.030	Hg	$L\beta_3$
				1.037	Tl	$L\beta_4$
1.129	W	$L\gamma_5$		1.038	Hg	$L\beta_2$
1.133	Ir	$L\beta$	Second order	1.038	As	$K\beta_2$
1.135	Ta	$L\gamma_1$	lines	1.038	Br	$K\alpha_1$
1.138	Ir	$L\beta_5$	possible	1.038	Au	$L\beta_5$
1.140	Pt	$L\beta_4$		1.042	Br	$K\alpha_2$
				0.664	Cb	$K\beta_1$
0.745	U	$L\beta_4$				
0.749	Cb	$K\alpha_2$	Third order	0.688	Zr	$K\beta_2$
0.752	Th	$L\beta_3$		0.700	Zr	$K\beta_1$
2.277	61	$L\alpha_1$	Theoretical value	2.073	61	$L\beta_1$

such a series the elements arrange themselves in the order of increasing atomic numbers, which would lead us to expect number 61 to crystallize between neodymium and samarium. Since both of these elements are relatively abundant, it would not be surprising to find these more common elements completely obscuring an intervening element if it were present in very small proportions. Consequently it seemed advisable to run a series using some other salt. Professor James has shown [6] that the solubility of the bromates decreases from neodymium to samarium, reaches a minimum in europium and then increases again in gadolinium and terbium. As a result these elements when fractionated in a bromate series do not arrange themselves in the order of

atomic numbers, but europium appears at the least soluble end while neodymium is well out toward the more soluble end with terbium, gadolinium and samarium distributed between.

When Dr. J. A. Harris fractionated some of our material as the bromate, it was found that samarium was crystallizing out at the less soluble end, while neodymium was concentrating in the more soluble fractions. As the fractionation continued the characteristic absorption bands of both neodymium and samarium

FIG. 1.


Wave lengths in Ångström's × 10
Logarithms of equivalent thickness.
Absorption bands in Visible Spectrum due to Neodymium
Absorption bands in Visible Spectrum due to Samarium
Absorption of fractions intermediate between Neodymium and Samarium, in which element it should concentrate showing impossibility of detecting new lines.

began to grow less evident in the middle fractions. Soon there was noticed at wave-length 5816 Å. a faint line which was growing stronger as the fractionation proceeded, while the characteristic absorption lines of both neodymium and samarium were growing weaker. Later another line was detected at 5123 Å. whose intensity increased as the familiar absorption lines of neodymium and samarium were becoming more faint. These lines evidently belonged to a new element whose presence had been concealed by the broad and persistent absorption bands of neodymium and samarium (Fig. 1). This unknown element we believe to be illinium, since our theories lead us to expect that such an element should exist at this point. When a bromate series is examined, europium and samarium appear in the least

soluble fractions followed in order by gadolinium, illinium, terbium and neodymium. This order is especially fortunate for the detection of illinium by means of absorption spectra because it is separated from samarium by gadolinium and from neodymium by terbium. Since the absorption bands of gadolinium fall in the ultra-violet portion of the spectrum and terbium has only a single narrow line in the visible spectrum, this leaves almost an uninterrupted field for the detection of any new absorption lines which might reveal the presence of a new element. As long as illinium remained associated with neodymium and samarium, its detection by absorption spectra was impossible. But as soon as it was sufficiently separated from these elements, the new absorption lines were quickly made out.

When the most prominent absorption lines of the elements of the rare earth group are plotted according to their wave-lengths [7] there is observed a drift (Fig. 2) in their positions. From elements 59 to 64 this drift is toward the shorter wave-length, the absorption bands of number 64, gadolinium, being in the ultra-violet region. When the new lines at 5816 Å. and 5123 Å. are assigned to element number 61, these lines fall into perfect accord with the drift in the lines of the neighboring elements. It is believed that this shift in the absorption lines must be intimately connected with the structure of the atoms which are responsible for the absorption bands. While the shift is less striking than that encountered in the study of the X-ray spectra of Moseley, it must be significant. A study of such phenomena should lead to a better understanding of the atomic structure of the members of the rare earth group.

In these days the identification of a new element is never complete without adequate measurements of its X-ray spectral lines and the determination of its atomic number. Accordingly [8] there was designed and built in our laboratory a glass X-ray tube equipment with water-cooled electrodes. The cathode was made of pure aluminum and the target of copper, the face being a plate with a depression in the centre for holding the fused rare earth oxide. The usual voltage was about 30,000 with an average current of 8–10 milliamperes. The equipment could not be used with safety above 40,000 volts, consequently we were unable to study the *K* emission spectra but were limited to the *L* series. We found it necessary to use exposures of 8–16 hours and

FIG. 2.

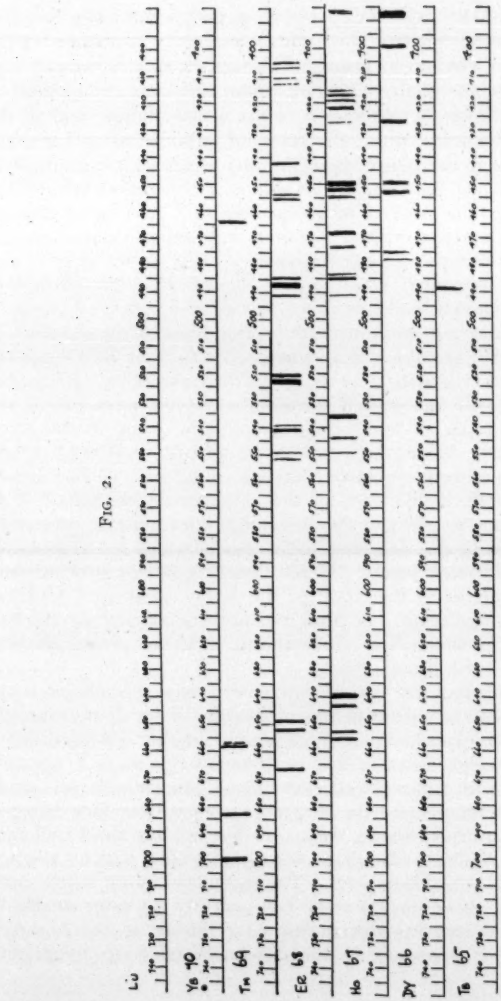
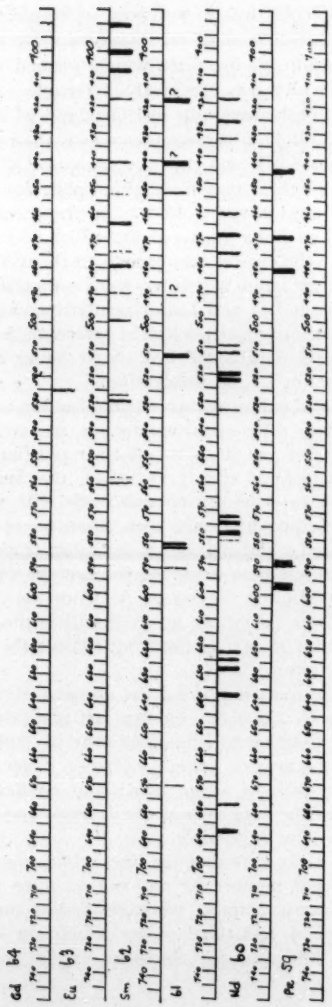


FIG. 2 (*Continued*).



Rare earth absorption spectra.

considerable difficulty was experienced in keeping our rare earth target in place during such a long exposure. By fusing the oxide into the depression on the plate which formed the face of the target, this difficulty was effectively overcome. Because of the skilful coöperation between our mechanician and our glassblower, we were able to obtain a perfect focus upon the rare earth button on the target, so that it was the only source of X-radiation. We were unable to get the copper lines on our plate for reference.

The crystal used in our work was calcite, three different crystals being used in order to avoid the effect of invisible defects. The rotation of the crystal was limited to thirty minutes because of the low energy input which we were compelled to use. The dispersion between $L\alpha_1$ and $L\alpha_2$ of samarium was 0.5 mm., the former line being our principle line of reference, but the deviation from the theoretical value of $L\alpha_1$ of 61 being checked against $L\alpha_2$ of samarium or $L\alpha_1$ of neodymium.

Five successful exposures were made for $L\alpha_1$ of element number 61, using two different fractions for the target. The lines obtained were faint, but in each case their positions were verified by several disinterested observers before the final reading was made. In every case the setting was made with the utmost care and after the reading had once been taken no re-reading of the line was permitted. The maximum deviation from the theoretical wave-length was less than 0.005 Å., the average value for the five determinations differing by 0.0004 Å. from the Siegbahn calculated value. These deviations are well within the limits of error of measurement as shown by checking against the lines of samarium and neodymium.

A single determination of $L\beta_1$ of element 61 varies from the calculated value by 0.004 Å. On our last plate there was a very faint line corresponding to $L\beta_3$ of number 61, but since this was at the edge of the area of general radiation covered by the oscillation, it is not safe to accept it without confirmation. Other measurements of the lines of both the *L* series and of the *K* series will be made as soon as possible.

In order to make sure that the lines which we have measured are due to element number 61 and not to some related element from our rare earth material, we have studied carefully all lines of the first, second, and third orders which are so situated that they might be mistaken for the lines of the missing element.

Table I shows these lines, with the wave-lengths, the elements and the series to which they are due. In addition a study of the relative intensities of these lines shows that about the only possibilities of mistaken identity would be in the case of chromium, cerium, lanthanum, mercury and bromine. Of these possibilities chromium could scarcely be present because no compounds of this element were used in our work and chromium is not a common constituent of rare earth minerals. Both cerium and lanthanum were present in the minerals from which our materials were extracted, but neither of these elements could have been present in our final samples because these were subjected to hundreds of fractionations after the last trace of the prominent praseodymium lines had disappeared and both cerium and lanthanum are farther removed from our materials than is praseodymium. The presence of mercury in our X-ray tube is scarcely possible because of the thorough evacuation and the presence at all times of a liquid air-trap between the pump and the tube. The presence of silver bromide on our photographic plate makes the presence of bromine absorption line possible, and on our last plate we had a rather sharp edge of radiation close to the $L\beta_1$ line of 61 which might possibly be due to the second order of the K absorption limit for bromine. The fact that both lines were distinguishable makes it highly improbable that our line was confused with the bromine line.

The three distinct lines of evidence which have been submitted are in complete accord with accepted theories, and it seems reasonable to conclude that such close agreement of fact and theory is significant. In order to justify the claim that a new element has been added to the Periodic Table, work is being pushed as rapidly as possible. The further investigation is centred along three distinct lines: (1) The finding of other ores which contain illinium; (2) the improving of methods of fractionation, including the use of other salts, and the elimination of previous losses; (3) a determination of the relation basicity of illinium. When these investigations have resulted in the production of more illinium and of material of much higher grade, it is expected that much information will be added concerning the youngest of the chemical elements.

Many other laboratories have been working along similar lines with much the same sort of materials. It is interesting to note the results reached by other investigators.

Prof. Wilhelm Prandtl, of Munich, has reviewed [9] carefully the evidence which has been published and finds that it is entirely inadequate for the establishment of the claim of the discovery of a new element. He is unwilling to accept the presence of strange lines in the arc spectrum of any rare earth material as significant in any way. He is not able to attach any importance to the finding of new bands in the absorption spectra. He criticizes the X-ray spectrum work because a vacuum apparatus was not used and because no attempt has been made to reproduce the X-ray lines on the printed page. Attention may be directed to the publication of Professor Prandtl [10] in 1924 in which he describes most careful work in a search for element number 61, the results of which were entirely negative. He therefore reached the conclusion that this element did not exist, or if it did exist, it could not be a member of the rare earth group.

Dr. Carl Auer von Welsbach has also called [11] attention to the work of Professor Prandtl and to his conclusion that there can be no such element as illinium in the rare earth group.

In November, 1925, Prof. C. J. Lapp, of the University of Iowa, read a paper before the annual meeting of the American Physical Society in which he reported [12] finding in the X-ray spectrum of material from the University of Illinois, a line whose wave-length corresponded closely with that predicted for $L\alpha_1$, of element 61.

Prof. J. M. Cork, working with the splendid X-ray equipment at the University of Michigan, gives [13] unmistakable evidence of the presence of element number 61 in the rare earth material which was so carefully purified by Profs. Charles James and H. C. Fogg, in the Chemical Laboratory of the University of New Hampshire. The publication of the *K* series lines is most convincing since these lines are much more characteristic than those of the *L* series of the rare earth group.

Messrs. R. J. Meyer, G. Schumacher and A. Kotowski, working [14] in their Berlin laboratory, have used both the bromate and the double magnesium nitrate methods of fractionation of their rare earths. In this manner they produced material from which they have obtained *K* series lines of illinium. This work is certainly of a very high order.

Prof. L. Rolla, of the University of Florence, worked upon a small quantity of didymium material obtained from Brazilian

monazite fractionating as the double thallium sulphate. He obtained evidence of the existence of a new element in the form of X-ray absorption spectra of the *K* series. A description of this work was deposited with the Italian Academy in 1924 in a sealed package. Since the announcement of the discovery of illinium this package has been brought to light and its contents published.[16] Because of this early work Professor Rolla claims priority for his discovery and insists that the element must be called florentium in honor of his city. In this connection it should be noted that the publication by the Bureau of Standards of the wave-lengths of 130 arc-spectrum lines is dated January 6, 1922, while five additional lines were added in an article[5] dated September 29, 1923. Professor Rolla began his work early in 1922, deposited his sealed package in June, 1924, and published it in November, 1926.

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Toxicity of Copper.—F. B. MALLORY, of the Boston City Hospital (*Jour. New Eng. Water Works Asso.*, 1927, **41**, 27–30), finds that copper compounds are a factor in the production of chronic hepatitis or cirrhosis of the liver. Copper compounds are absorbed when ingested. They then produce hemolysis, that is, liberate the hemoglobin or blood pigment from the erythrocytes or red blood-cells. If the rate of liberation of hemoglobin exceeds the rate at which it is excreted by the kidneys, that pigment is changed to hemofuscin, a light-yellow pigment, which deposits in the liver. Both hemoglobin and hemofuscin contain iron, which is in organic combination and does not yield iron cations. In the course of time, the hemofuscin is changed by the liver cells into an orange-yellow pigment, hemosiderin, which contains iron and yields iron ions. These derived pigments (hemofuscin and hemosiderin) exert a poisonous action upon the liver cells and finally destroy them, meanwhile an increase occurs in the connective tissue which supports the liver cells and holds that organ together. With repeated small doses of copper compounds, an animal may live for years. A lethal or fatal dose of a copper salt produces death by blocking the kidneys.

Copper may gain entrance to the body in several ways. Foods, especially fruits, which have been cooked in copper vessels, contain salts of that metal. The various organic acids, such as acetic, malic, citric, lactic, and oleic, attack the vessel and dissolve copper. Distilled liquors may contain copper salts, formed by the action of their organic acids. These acids are present in the mash, accompany the alcohol during distillation, and attack the copper worm of the condenser. In a series of almost 2000 samples of liquor, copper was found in over 10 per cent. of the samples in amounts ranging from a trace to 69 milligrams per litre (69 parts per million). In another series, consisting of fewer samples, one liquor contained 185 milligrams of copper per litre. Among workers, who are exposed to dusts containing copper, that element may gain entrance to the body through either the mouth or the lungs.

Another disease, hemochromatosis, may be due to the injurious action of copper on the organism. It is characterized by changes in the liver similar to those described above, and by bronzing of the skin. The pancreas may also become involved, and diabetes mellitus occur. J. S. H.

New Source of Arsenic in the Human Environment.—ROE E. REMINGTON, of the North Dakota Agricultural College (*Jour. Am. Chem. Soc.*, 1927, **49**, 1410–1416), finds that samples of American smoking and plug tobacco contain from 6 to 30 parts of arsenic per million. Approximately one-half of the arsenic in pipe tobacco is evolved in the smoke, and approximately one-half of the arsenic in plug tobacco is soluble in water. These amounts of arsenic are in excess of those normally present in plants and animals, and also in excess of the amounts permissible in foods. J. S. H.

THE PHYSICAL REALITY OF LIGHT-QUANTA.*

BY

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To MY keen regret I am prevented by pressing duties from receiving in person the high honor which The Franklin Institute has conferred upon me. Accordingly I ask permission to lay before you as a token of my profound gratitude a short communication upon a scientific question which at the present time occupies the foreground of interest.

You all know that theoretical physics, which developed and progressed consistently through two hundred years, and only a generation ago seemed near to its final conclusion, has now entered a critical period, fraught with serious consequences. Not that its fundamental principles have been questioned! For its most general and at the same time its simplest laws, such as the Principle of the Conservation of Energy, the Laws of Thermodynamics, and the Fundamental Equations of the Electromagnetic Field, are just the ones which have so far withstood successfully the most severe trials, and serve now as ever before as the guides for wider exploration. It would then be entirely incorrect to speak of a breakdown of the science. It is our mental pictures of the occurrences, in which these principles find their application, that in recent times have been thrown into confusion. Many concepts which could be counted among the simplest and the most evident in the world, have turned out to be obscure, doubtful, even contradictory, and it is certain that in many respects we must straightaway build up from the very beginning, if we are not to lose sight of the most important assumption of physical research, the compatibility of the various laws with one another.

For example, no occurrence in nature appears more simple and more obvious, alike to the layman and to the investigator, than the motion of a material body, a sphere for instance, and to this simplicity of ideas corresponds completely the simplicity of the

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laws which so far have been recognized as governing the motion. It is no wonder that man has sought for ages, even from the time of the Greek philosophers, to reduce all physical occurrences to the motions of material bodies, and that he was later on strongly confirmed in this endeavor by the success of the brilliant discoveries of Galileo, Kepler, and Newton.

To-day we know with certainty that the laws of mechanics possess only an approximate validity. I am not referring now to the emendation brought about by the Theory of Relativity. For, although this theory has affected our views fundamentally, nevertheless in the last analysis it means not a complication, but on the contrary a simplification and a refinement of the classical mechanics. What we are now considering has to do with something entirely different, something far more revolutionary. That is to say, experience has obliged us to recognize the inevitable conclusion, that not only are the laws of mechanics not fundamental, but that even the basal concept of mechanics, the material particle, under the circumstances of sharply curved motion, immediately loses its meaning. When a particle, an electron, for example, moves in this way, it means nothing to speak of a definite position which the electron occupies at a definite time. The more sharply the path curves, the more blurred becomes the position of the electron. It becomes indefinite and, so to speak, spreads out into surrounding space, much as a beam of light, when striking the edge of a screen, instead of continuing forward as a unit, bends and scatters in all directions. If the path of the electron is a periodic or a quasi-periodic one, and if it extends over a very small amount of space, as in Bohr's model of the atom, then at every instant the electron spreads over its entire path, and its motion resembles rather the vibrations of a standing wave in a continuous medium than those of an oscillating particle.

Thus corpuscular mechanics resolves itself into an undulatory mechanics, the principles of which in all their details have not by any means been fully investigated. Still, thanks to the ideas introduced into science by L. de Broglie and E. Schroedinger, these principles have already established a solid foundation. We may place confidence in them all the more because their consequences agree completely with the mathematical postulates which had been introduced earlier into quantum mechanics by W. Heisenberg, solely upon the basis of facts of experience.

If thus at least a prospect is offered of gradually attaining a deeper insight into the true nature of mechanical energy, still the way toward an understanding of the nature of the energy of electromagnetic radiation seems at the present time to be completely closed. Here we are experiencing in a certain sense a development just the opposite of that in corpuscular motions. While the corpuscular quanta, as we have seen, in sharply curved motions spread out in space and dissolve into wave-forms, it appears on the contrary that radiant energy travelling in absolute vacuum with the speed of light, at high frequencies shrinks together and concentrates itself at separate points which move like corpuscles, and for that reason are called light-quanta.

At the first glance this latter state of affairs must appear far more incomprehensible than the former. For in corpuscular motions we have to do with matter, or with electric charges, and these surely still conceal within themselves so much of the mysterious that therewith may be associated many a riddle, whose solution will only be found when we shall have succeeded in rending the veil of the mystery. But the laws of propagation of radiant energy in absolute vacuum, we had the right to regard as known in all their details since the brilliant success of Maxwell's theory. Absolute vacuum surely conceals within itself no mystery, no matter, no electric charge. It serves only as the carrier of the electromagnetic field. And the laws of this field are represented with a completeness and an exactness, which hold their own against the finest interference measurements, by equations to which a concentration of energy into quanta is entirely foreign. For the elementary quantum of action plays no part in Maxwell's equations. From dimensional considerations it would be entirely impossible to introduce this quantity into Maxwell's equations unless additional constants should appear therein.

Just at the point, then, where conditions seem to be the simplest, and where, according to all previous experience, we had the right to feel ourselves nearest to a final comprehension of Nature, we are baffled by an entirely unexpected mystery. Again and again the question arises, must we really ascribe to the light-quanta a physical reality, or is there after all a way of taking account of them, which preserves the validity of Maxwell's classical electrodynamics?

Many have been the endeavors to answer this question, and still the argument surges to and fro. Let us now give the question a brief consideration.

To begin with, it is clear that a decision can only be reached by the closest attention to the facts of experience. Since in the last analysis we can never measure the electromagnetic field itself, but only its effects upon matter, that is upon measuring instruments, it might at the first glance appear promising to limit the significance of light-quanta solely to the interaction between radiation and matter, that is to the processes of emission and absorption, and on the other hand to deny their existence in the propagation in absolute vacuum. Then all the classical laws of the radiation of energy in vacuum could be retained.

But a closer consideration will show us that this way out leads nowhere if we are to hold fast to the ever trustworthy fundamental principles of physics. First of all, it cannot be doubted that a physical reality must be ascribed to radiant energy in vacuum, as such. This follows from the First Law of Thermodynamics, the Energy Principle, in its application to the emission and absorption of radiant energy. But not only does a beam of radiation possess a definite energy according to the First Law, it has also a definite entropy according to the Second Law, the Principle of the Increase of Entropy. For if the entropy were not present it could not increase. We are thus obliged to ascribe to entropy, just as to energy, an independent existence, without reference to any matter whatever. This conclusion is in no wise affected by the fact that, in order to determine the quantity of entropy, we must measure the temperature of a material body which stands in equilibrium with the radiant energy.

If we now retain also the relationship between entropy and probability introduced by L. Boltzmann, and without it an understanding of the content of the Second Law seems impossible, there follow laws of the fluctuation in space and time of the energy in a beam of radiation of definite temperature.

If we now compare the law of energy-fluctuation deduced from the measured entropy of radiation with the law of fluctuation called for by the classical theory, we find that in addition to the fluctuations yielded by the latter law, there appears a new and entirely different kind of fluctuation, the statistics of which can only be explained by the presence of discrete atoms of energy of

the magnitude of light-quanta. For the fluctuations are much too great at low temperatures to be accounted for by the classical theory.

In this entire study the interaction between radiation and matter plays no part whatever. We then cannot avoid ascribing to light-quanta in absolute vacuum a real physical existence. This was pointed out as early as 1909 by A. Einstein. On the other hand, however, we may not look upon light-quanta as independent of one another. For then we would obtain only the second kind of energy-fluctuation, and not those fluctuations which predominate at high temperatures, and are yielded by the classical theory. These, too, are called for by the law of radiation based upon measurements.

Here opens then that break which in my opinion penetrates to the depths of the structure of the quantum theory. None of the more recent advances has filled it up. Beyond doubt the statistical interdependence of light-quanta is related to the interference phenomena of light rays from the same source. One might perhaps think of it in this way, that each light-quantum carries about with it, as it were, a mark of its origin, and that two light-quanta of like origin can interfere with each other when they meet. But the difficulty would not be avoided by thinking in this way. For at feeble intensities the chances of an encounter are much too small to account for those interference phenomena which actually occur. Rather do Maxwell's equations of the field, which do not involve light-quanta at all, appear to represent the interference phenomena completely and exactly, even down to the feeblest intensities of light.

For these reasons we shall find it impossible to think of the energy of light-quanta as concentrated at separate points in space. Rather there goes out from each light-quantum a kind of action-at-a-distance, and indeed not only at a distant place but also at a distant time, for according to the Theory of Relativity we cannot distinguish in this connection between space and time. In fact, the form of many very general principles of general mechanics and atomic physics brings out the thought that the course of an event is dependent as much upon the final state as upon the initial one. In this way is introduced a certain direct interaction between the two states separated from one another in time. The Principle of Causality would be influenced thereby only in its form, not in

its substance. For all that, such trains of thought mean a difficult undertaking for our present-day powers of conception. To carry them out would bring about far-reaching changes in all our views of the physical world.

What is there to be gained which makes such a great sacrifice seem worth while? It would be rash to be willing to express a judgment at the present time. Still I would like to make at least an attempt to point out the direction in which the end sought for may lie. Probably we might catch a glimpse of it in the complete consolidation of the two great regions of physics which still are separated by an impassable chasm, corpuscular physics and physics of the continuum, or wave-physics. If the goal shall some day be attained, these two regions will no longer appear fundamentally different from one another but will represent only the opposite ends of a single region which includes them both.

The classical theory recognizes and treats only the two extreme cases; on the one side, corpuscular motions, on whose outermost border lies the uniform motion of a particle in a straight line; on the other side, wave-motions, on whose outer limit lies the static, homogeneous field. Looked upon from the newly established point of view, there is neither pure corpuscular motion, nor any pure wave-motion. Rather, every corpuscular motion includes something of wave-motion, and every wave-motion something of corpuscular motion. The difference is only gradual and quantitative. In the motion of a particle, as soon as the ratio of the impulse to the curvature of the path, which in motion in a straight line has an infinite value, sinks to the order of magnitude of the universal constant of action, the laws of wave-motion begin to play an appreciable part. And *vice versa,* in monochromatic light, as soon as the ratio of its energy to its frequency, which is infinite in a static field, sinks to the order of magnitude just mentioned, the corpuscular laws begin to be appreciable. In what relation, however, the corpuscular laws stand to the laws of wave-motion in the general case, remains the great problem, to which at the present time a whole generation of investigators is devoting its best efforts. We can entertain no doubt that finally a satisfactory solution will be found, and that then theoretical physics will have made another significant advance toward the attainment of its ultimate goal, the building up of a unified system embracing all physical phenomena.

THE SUN AS A RESEARCH LABORATORY.*

BY

GEORGE ELLERY HALE, Sc.D., Ph.D., LL.D.

Franklin Medallist; Member of the Institute.

THE award of the Franklin Medal, so closely following your generous gift of the Elliott Cresson Medal a year ago, is a great and unexpected honor, for which I cannot adequately express my appreciation. Needless to say, it would be a privilege and a pleasure to offer my thanks in person, and only the express orders of my physician prevent me from doing so. I am indebted to my friend and former colleague, Prof. Harlow Shapley, for his kindness in representing me at the Medal Day exercises.

Without entering into the details of my life work, I cannot forbear to say that any results obtained have been largely due to the exceptional privileges and advantages I have enjoyed since my boyhood. Chief among these have been the strong encouragement and generous help of my father and mother, my wife, and other members of my family, and of many friends and colleagues. Credit for work thus accomplished cannot, of course, be claimed solely by myself, as it belongs in such large measure to those with whom I have had the good fortune to be associated. I therefore consider your award of the Franklin Medal as a recognition of several joint undertakings in which I have had a part.

In choosing a subject for the paper requested by your Secretary, I have thought this an opportune occasion to speak of the value of solar research to the physicist and chemist, who have derived from astronomical sources, of which I can mention only a few, some of the most important principles and methods employed in their investigations of the constitution of matter. Thus modern industry, from which astronomy may seem so remote, is actually indebted to cosmic laboratories for the solution of problems that have been vitally important to its progress.

Another point I wish to emphasize is the opportunity for fresh discovery that now awaits the investigator, whether amateur or

* Presented at the Stated Meeting of the Institute held Wednesday, May 18, 1927. Read in the absence of Doctor Hale by Prof. Harlow Shapley, Harvard University.

professional, who attacks the sun with the aid of methods and ideas chiefly due to recent advances in physics.

We may think of the sun in several different aspects. To a dweller on the earth it is of prime significance as the central body of the solar system, whose varying activity can be detected, not only by changes in its surface and atmosphere, but also by variations in its total radiation and by the auroras, terrestrial magnetic storms, earth currents in telegraph and cable lines, and disturbances in radio reception which are traceable to its influence. From another standpoint we may regard the sun as a typical star, the only star near enough the earth to be examined in detail, and thus invaluable in the study of stellar evolution. Finally, we may think of the sun as a research laboratory, in which fundamental problems of physics and chemistry are open to solution. It is chiefly in this last aspect that I shall consider the sun to-day, though I shall also refer to it briefly from other viewpoints.

The conception of the sun as a vast physical and chemical laboratory, in which we may watch the progress of experiments beyond our limited capacity to perform, is not a new one, but our appreciation of its possibilities has been greatly enhanced by recent progress in science. We now realize that within the sun, where the temperature reaches millions of degrees centigrade, the atoms are wholly or partially stripped of their electrons. Between this internal region and the visible surface of the photosphere, the normal chemical elements are formed and we can observe their vapors, at a temperature of about 6000° C., in the lower part of the solar atmosphere, which in sun-spots is cool enough for certain chemical compounds to exist.

As the sun contains most of the familiar chemical elements, it is not surprising that this high-temperature furnace provided the necessary means of laying the foundations of spectroscopy. This first essential step in the recent determination of the true nature of matter was taken in 1859 by Kirchhoff and Bunsen, who compared the light emitted by metals vaporized in their laboratory with that emitted by the vapors glowing in the laboratory of the sun. When analyzed by their prisms, both light-sources were found to contain many coincident lines, thus proving the presence of various elements in the sun. The solar lines appeared dark, showing that the white light of the hot solar disc

was characteristically absorbed by the cooler vapors of its atmosphere. Thus the spectroscope took its place, both as a means of chemically analyzing celestial objects and tracing the course of their evolution and as the chief guide in the great research which is now revealing the structure of the atom.

The solar spectrum, when first examined, seemed an orderless jumble of thousands of lines. Apparently scattered without rhyme or reason, many of them could be identified one by one with their terrestrial counterparts, but the process was purely empirical. The first sign of the marvellous order which has recently replaced chaos came when Huggins photographed the spectra of the white stars. There, in obvious sequence, widely separated in the visible spectrum but rapidly converging toward a limiting head in the ultra-violet, lay the beautiful series of hydrogen lines. Easily represented by a simple formula, they pointed the distant way toward the rational knowledge of complex spectra which has resulted from Bohr's theory of atomic structure.

The next significant step in the same direction was also due to the study of celestial spectra. When Lockyer applied a spectroscope, attached to an equatorial telescope, to the analysis of different parts of the sun, he was greatly impressed by the peculiar variations in the relative intensities of certain lines, notably those of calcium. The pair of calcium lines at the beginning of the ultra-violet, known as *H* and *K*, strangely contrasts in behavior with the blue calcium line in various solar and stellar spectra. Such differences led Lockyer to the discovery of "enhanced" lines, which play such a fundamental rôle in modern spectroscopy.

Prof. Charles A. Young, first of Dartmouth and later of Princeton, whose eyes were unusually sensitive to light in the extreme violet, was the first to observe that the *H* and *K* lines reach to the top of the highest prominences, which sometimes rise more than 400,000 miles above the sun's surface. The significance of this fact is best appreciated when we recall that the only other lines that attain these immense heights in the solar atmosphere are those of the lightest known gases, hydrogen and helium, while the strong blue line of calcium, conspicuous in the solar spectrum, is confined to low levels and does not accompany the *H* and *K* lines in quiescent prominences.

Recent knowledge of the atom, applied to this problem by Saha, has shown that the *H* and *K* lines are due to ionized cal-

cium, which has lost one electron, while the blue line represents the complete calcium atom. If time permitted I might point out the beautiful applications of modern atomic theory, not only to the explanation of many solar and stellar phenomena, but as the chief clue to our present understanding of the evolution of the stars.

Another notable case where solar conditions permitted a fundamental chemical discovery was Lockyer's detection of helium in the chromosphere and prominences. In his first study of their spectra in 1868 he noticed a brilliant yellow line, which he attributed to an unknown gas. This he named helium, and for many years it seemed to remain the exclusive property of the sun and of certain stars. In 1896, however, a light gas extracted from uraninite by Ramsay was found to give a yellow line, apparently in the same position. Ramsay's low-dispersion spectroscope was quite inadequate to settle the question, and he sent a sample of the gas to Runge. When examined with a powerful concave grating the yellow line fell in the right position, but it had a faint close companion and was also accompanied by several more refrangible lines of the same character. This led to the discovery that the yellow helium line in the sun is also double and to the detection of the other helium lines in solar, stellar, and nebular spectra.

We have thus seen how solar observations, after serving Kirchhoff and Bunsen as the means of establishing the fundamental principles of spectroscopy, soon resulted in the discovery of a new and important element and gave in the behavior of the "enhanced" lines the first clue to the modern theory of the constitution of matter. Let us now glance at a remarkable experiment performed for us by every sun-spot.

In 1862 Faraday made the last experiment of his long and eventful career. He had previously discovered the effect of a magnetic field on polarized light passing through it. He now wished to learn whether the light emitted by a radiating atom could be altered in any way by the influence of magnetism. Placing a sodium flame between the poles of a powerful electromagnet, he watched the lines of its spectrum when a current was passed through the coils. But no change whatever could be detected—the lines appeared precisely the same with and without the field.

In 1892 Professor Young found that several lines which were

single on the sun's disc were double in certain sun-spots. In harmony with the knowledge of the time, he supposed these spot-lines to be "reversed" by the absorption of a cooler gas above them. As a matter of fact, he was observing the very effect that Faraday had sought in vain, though this did not appear until many years later.

Professor Zeeman, in his Franklin Medal address, has described the steps leading to his famous discovery of 1896, when he found that many spectral lines are split into two or more components, polarized as predicted by Lorentz's theory, when the luminous source is observed in an intense magnetic field. It was thus a comparatively simple matter, at the Mount Wilson Observatory in 1908, to apply the necessary tests for polarization to the sun-spot lines and thus to prove their doubling to be due, not to other possible causes which had previously seemed sufficient, but to the presence of a magnetic field in every sun-spot. Perhaps it should be added, however, that two series of preparatory investigations preceded these tests: (1) The development of the spectroheliograph and its application to the study of higher and higher levels of the solar atmosphere, resulting in the discovery in 1908 of great vortices in the hydrogen atmosphere surrounding sun-spots; (2) a photographic study of sun-spot spectra at the Kenwood, Yerkes and Mount Wilson Observatories, leading to the design of the 60-foot tower telescope and its powerful subterranean spectrograph, with dispersion sufficient to resolve photographically the double lines in the spots.

The Zeeman effect is shown by thousands of lines in the sun-spot spectrum and the complete tests involve the study of their plane and elliptically polarized components by methods which cannot be described in detail here. The most striking bit of evidence may, however, be of interest. In the laboratory, with certain arrangements of the polarizing apparatus, the right component of a line doubled by the magnetic field, when observed in the direction of the lines of force, appears alone, without the left component. Then, when the direction of the current is reversed through the coils of the magnet, the right component vanishes and the left one appears alone. The analogous case in the sun should occur in two single spots, lying near the central meridian in the northern and southern hemispheres, with electrified particles whirling in opposite directions in their respec-

tive vortices. Two such spots chanced to be thus placed in September, 1908, and it was no small pleasure to find that two photographs of their spectra, taken with the polarizing apparatus unchanged, showed the right component of the Zeeman doublets in one spot and the left component in the other.[1]

Passing over the application of this method to the determination of the peculiar law of sun-spot polarity, let me refer briefly to another interesting experiment in magnetism performed for us by the sun.

Schuster long ago raised the question whether all rotating bodies are magnets and Einstein has recently given serious study to the same problem.[2] The earth is a great magnet, but while its rotation is the probable cause of its magnetism, it is difficult to prove this conclusively. It would evidently be advantageous to learn whether other great rotating bodies, such as planets or stars, are also magnets. Thus far the sun appears to be the only celestial object suitable for the experiment. Its great mass and large disc, and the presence of luminous vapors in its atmosphere, permit the Zeeman test to be applied in both the northern and southern hemispheres. Even with the high dispersion of a 75-foot spectrograph, in the second order spectrum of a large Michelson grating, the effect is extremely difficult to detect, because the general magnetic field of the sun is so much weaker than the fields observed in sun-spots. But the minute line-displacements observed at various latitudes with special polarizing apparatus consistently unite in showing the existence of this field, and in proving that its polarity, as would be expected because of the same direction of rotation, is the same as that of the earth's field.

The three crucial tests of the Einstein theory are beyond the capacity of terrestrial laboratories. In determining whether a ray of light can be deflected from a straight line by a gravitating mass, even a body as large as the earth would be altogether inade-

[1] As I cannot enter here into a discussion of the complex nature of the superposed sun-spot and hydrogen vortices, I must refer to such papers as "The Law of Sunspot Polarity," *Astrophys. Jour.*, **52**, 270–300, 1925, and to an article in *Nature* for May 14, 1927, on "The Fields of Force in the Atmosphere of the Sun." St. John's important paper on "Radial Motion in Sun-spots," *Astrophys. Jour.*, **37**, 322–353, 1913, should also be consulted.

[2] See also the investigations of Barnett.

quate for the purpose. Einstein pointed out, however, that the great mass of the sun should be sufficient to cause an easily measurable change in the apparent position of a star lying near its circumference. Repeated determinations of such deflections, made at total solar eclipses by the ablest astronomers, show their almost perfect agreement with calculation and afford the strongest proof of the validity of Einstein's theory.

His theory also calls for a shift toward the red of the lines of the solar spectrum, which has been measured by Evershed, St. John, and other solar physicists. The latest and most complete investigation of this question, which we owe to St. John, brings out its intricacy but leaves no doubt as to agreement with Einstein. The prediction by Eddington that the companion of Sirius is some 50,000 times as dense as water, calling for the large displacement of the lines of its spectrum actually found with the 100-inch Hooker telescope by Adams, is the most striking confirmation of the validity of Einstein's theory and of modern views on the constitution of matter and the evolution of the stars.

These few illustrations, which might be supplemented by many others, should leave no doubt regarding the value to the physicist and chemist of the research laboratories afforded by the sun and stars. Let me now turn to another class of solar phenomena, the study of which should aid in the solution of various geophysical problems.

I have referred more than once to Prof. Charles A. Young as a pioneer of solar research. It is a pleasure to add that we are indebted to him for the essential principle of the spectro-helioscope, an instrument which has recently brought many extraordinary phenomena of the solar atmosphere within the range of observation.

The spectroscope was first applied to the observation of the solar prominences in 1868. Janssen, following a solar eclipse in India, and Lockyer, working independently in England, found almost simultaneously that the bright light of the sky around the sun, which hides these immense red flames from us, can be weakened in any desired degree by dispersion. The prisms or grating of a spectroscope draw out this scattered sunlight into a long rainbow band, on which the bright lines of hydrogen, helium, and calcium, which are not weakened by dispersion, become plainly

visible. If the narrow slit is radial to the solar image, the height of any part of a prominence projecting beyond the sun's circumference can be determined by measuring the length of one of these bright lines, but the form of the prominence cannot be seen. Several astronomers suggested that the slit be made to oscillate across the image of the prominence, hoping that the oscillating bright line would reveal the form by persistence of vision. They forgot, however, that the oscillating continuous spectrum would blot out the comparatively faint image of the prominence. Young recognized the need of a second slit, in the plane of the spectrum, narrow enough to exclude all light except that of the bright line and arranged to oscillate precisely with it. A pair of oscillating slits, attached to his solar spectroscope, is illustrated in the JOURNAL OF THE FRANKLIN INSTITUTE for November, 1870. With this instrument, attached to the Dartmouth telescope, Young was able to see the forms of the prominences projecting beyond the edge of the sun.

At this juncture Zöllner and Huggins found that by widening the fixed slit of a spectroscope, preferably placed tangent at its inner edge to the solar circumference, the form of a prominence could be seen with the hydrogen lines as through an open window. Young, who had been troubled by vibration of his telescope caused by the rapid oscillation of the slits, at once adopted the fixed wide slit and discarded the oscillating principle, which remained unused for half a century.

Shortly after the discovery of the hydrogen vortices with the spectroheliograph, it struck me that they might perhaps be rendered visible by the use with our 30-foot spectroscope of a rotating disc, bearing a large number of radial slits. This method, which embodies in a somewhat different form the principle used by Professor Young, is perfectly sound, but in my early experiments the monochromatic image was too faint to be distinctly seen, and I was prevented by the demands of other work from developing the apparatus at that time. In 1924 I built a spectrohelioscope (as I have named it), comprising two sets of five slits each, carried at the opposite ends of an oscillating bar, mounted on a bearing half way between the optical axes of the collimator and observing telescope of a reflecting spectroscope of 13 feet focal-length. A two-inch solar image was formed on the first slits and the monochromatic hydrogen image was viewed through

the second slits by means of a low-power eye-piece. In the present form of the apparatus, which is mounted vertically in my solar laboratory in Pasadena, two adjustable single slits are used instead of the two sets of multiple slits. The light aluminum bar that carries them is supported on an accurately made bearing, like that of a jeweler's lathe, and is rapidly oscillated by means of a small electric motor. The best dispersion for the hydrogen line $H\alpha$ is that of the bright first order spectrum of a grating ruled with about 15,000 lines to the inch.

This instrument affords superb views of the hydrogen flocculi, including the finest details of the vortex structure shown on the best Mount Wilson spectroheliograms. An account of the new results thus far obtained may be found in a paper in *Nature* for May 14, 1927. While the spectroheliograph remains the best means of photographing the phenomena of the solar atmosphere, it is greatly surpassed by the spectrohelioscope in two particulars: (1) The ready detection of flocculi moving with great radial velocity, which displaces their $H\alpha$ line from the second slit of the spectroheliograph and may thus prevent them from being recorded photographically, and (2) the possibility of measuring the radial velocity of any such active region. The simple device used for these purposes is merely a plate of plane parallel glass mounted below the oscillating second slit on a spindle easily rotated by a milled head, and bearing a graduated arc. When the plane of the glass is parallel to that of the oscillating slit, the $H\alpha$ line is not displaced. By rotating the glass, the line is displaced toward red or violet, thus bringing a rapidly moving mass of hydrogen into view. The graduated arc, when properly calibrated, gives the radial velocity of the hydrogen.

On January 24 and 25, 1926, I observed with the spectrohelioscope near the centre of the sun a very brilliant flocculus, of great area and rapidly changing form. Two or three weeks later I received from Prof. Carl Störmer, of Oslo, a letter describing the most brilliant red aurora (of January 26th) he had observed in years, and asking whether I had seen anything remarkable on the sun. An intense magnetic storm accompanied this aurora, and both were probably due to the discharge of electrified particles marked by the brilliant solar outburst I was fortunate enough to see.

The spectrohelioscope thus supplies a simple means of adding

to our knowledge of the true relationship between solar and terrestrial phenomena. Pickard has recently shown a definite correlation between solar activity and the phenomenon of fading in radio. It is probable, however, that this correlation could be materially improved by measuring the sun's activity, not in terms of the total area or number of sun-spots, but rather by the area and brightness of the bright hydrogen flocculi. Brilliant outbursts, such as that mentioned above, almost invariably occur in the regions surrounding active sun-spot groups rather than in the spots themselves. Quiescent sun-spots, even those of great size, may cross the sun's disc with little, if any, effect on the aurora or the magnetic field of the earth. Thus it is desirable to organize a continuous watch of the sun, to record all exceptional outbursts and especially to learn exactly when they begin.

To aid in this undertaking, I have devised an inexpensive solar telescope and spectrohelioscope, the mechanical parts of which can be made for about three or four hundred dollars. If satisfactory grating replicas can be made, the total cost of the optical parts will be considerably less than that of the mechanical parts. Radio amateurs and others equipped with the necessary tools could easily build the mechanical parts for themselves. I hope that in the near future at least eight spectrohelioscopes, well distributed around the world in longitude, may be used in conjunction with the few available spectroheliographs in maintaining a nearly continuous watch of the hydrogen atmosphere of the sun.

THE REFRACTION OF X-RAYS.

BY

BERGEN DAVIS, Ph.D.

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IMMEDIATELY following the discovery of X-rays many experiments were performed by Röntgen and by others to determine the nature of the new phenomenon. It was sought to determine whether they were a stream of minute particles or an electromagnetic radiation of the nature of light. If of the nature of light, they might be expected to show the properties of light such as the capacity to be reflected and refracted. Attempts, however, to detect either reflection or refraction of these rays were unsuccessful for many years.

The most searching experiments on refraction were made by Chapman and Barkla [1] soon after the discovery of crystal reflection, of the line spectra and the establishment of the fact that these rays were of the same nature as light.

The experiments of Barkla were made with a prism of bromine vapor, using an X-ray frequency near the k resonance frequency of bromine. He concludes that if refraction takes place the value of $\delta = 1 - \mu$ is less than 6×10^{-6} (where μ is the refractive index). This is the order of refraction that has since been observed.

REFRACTION AT SURFACE OF A CRYSTAL.

Following the discovery of crystal reflection in 1912 by M. Laue, Sir William Bragg showed that

$$n\lambda = 2d \sin \theta$$

expressed the relation between the wave-length of the radiation, the grating space of the crystal and the angle at which the rays are reflected. We at once see that

$$\frac{\lambda}{2d} = \frac{\sin \theta}{n}$$

should be a constant for all orders of reflection.

[1] CHAPMAN, *Proc. Camb. Phil. Soc.*, p. 374 (1912). BARKLA, *Phil. Mag.*, 31, 257 (1916).

Refinements in measurement of wave-length soon showed that this ratio was not quite constant. This departure from constancy was attributed by Stenstrom [2] to a slight bending (refraction) of the rays on entering and leaving the crystal surface.

If the rays make an angle θ_1 with the surface at entrance and an angle θ_0 after entrance, then since the molecular planes are parallel to the surface, the refraction is given by

$$\mu = \frac{\cos \theta_1}{\cos \theta_0}.$$

The true wave-length is represented by

$$n\lambda = 2d \cos \theta_0,$$

while the angle θ_1 is the one actually measured.

The departure of the index μ from unity is

$$\delta = 1 - \mu = 1 - \frac{\cos \theta_1}{\cos \theta_0}.$$

For orders m and n this may be written

$$\delta = \frac{\left(\frac{\sin \theta_m}{m}\right)^2 - \left(\frac{\sin \theta_n}{n}\right)^2}{2\left(\frac{\cos \theta_m}{m}\right)^2 - 2\left(\frac{\cos \theta_n}{n}\right)^2}.$$

Actual measurements of wave-length by the photographic method at various orders by Stenstrom were qualitative in that they showed a progressive change with order, but reliable values of the refraction δ were not obtained. Davis and Terrill,[3] using the ionization spectrometer method, were not able to determine δ within 100 per cent. of the correct value. The refractive effect is very small. The bending of Mo_1K_α radiation entering calcite crystal at angle of reflection is about 3″ of arc, which is near the limit of measurement.

This small refractive bending was greatly increased by cutting the surface of the crystal to an angle φ to the reflecting planes.[4, 5] The crystal is so cut that as the ray enters the crystal at the

[2] STENSTROM, "Dissertation," Lund (1919).

[3] DAVIS and TERRILL, *Proc. Nat. Acad. Sci.*, **8** (1922).

[4] DAVIS and HATLEY, *Phys. Rev.*, **23**, 290 (1924).

[5] DAVIS and VON NARDROFF, *Proc. Nat. Acad. Sci.*, **10**, 2 (1924).

proper angle for reflection from the planes it makes a very small angle with the surface. (Fig. 1.) The bending at the surface is greatly increased by this device. As an illustration in the case of the reflection of MoK_β radiation from pyrites, the bending in case of uncut crystal was 3″ of arc, and in case of the crystal cut at angle to the planes it was 160″ of arc.

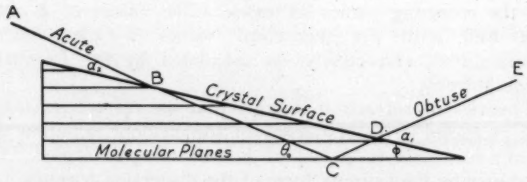
It may be shown for a crystal cut so that the surface makes an angle φ with the reflecting planes that

$$\delta = \frac{(\sin \alpha - \sin \theta_0)\ (\sin^2 \alpha - \sin^2 \varphi)}{\sin \alpha \cos^2 \alpha},$$

where $\alpha = \frac{1}{2}$ (180° − angle turned through) in going from position where rays pass in one direction to position in which they traverse the crystal in opposite direction.

A specially designed spectrometer was used for these measure-

FIG. 1.



Reflection of a beam from a crystal ground at an angle to the molecular planes.

ments. No readings of angle were made with verniers. The instrument was provided with an optically calibrated tangent screw and hand-wheel. The measurement of turn of crystal through 180° was also made by an optical method. A plane parallel mirror, plane and parallel on both sides, was mounted above the crystal. A distant telescope and scale permitted one to turn the crystal through 180° with an accuracy possibly as great as one second of arc.

Measurements of the refraction of MoK_α ($\lambda = .7078$) in calcite gave a value of $\delta = 2 \times 10^{-6}$, while the calculated value is 1.9×10^{-6}.

The results of a series of measurements in pyrites are given in Table I.

TABLE I.

$\lambda = 0.7078$.

φ.	$\alpha = \frac{1}{2}(180° - \beta)$.	$\theta_1 - (\theta_0 - \varphi)$.	δ ($\times 10^6$).
0°	7° 31′ 28″	3.6″	4.6
6° 31′ 57.5″	7° 31′ 43.5″	39″	3.26
7° 18′ 39″	7° 32′ 48.5″	159″	3.37

$\theta_0 = 7° 31′ 22.9″$.

$\lambda = .6311$.

0°	6° 42′ 18.5″	3″	3.87
6° 31′ 57.5″	6° 43′ 41″	160″	2.82

$\theta_0 = 6° 42′ 13.4″$.

φ = angle between crystal surface and planes.
β = angle turned through from position A to B.

The angle of bending of the rays on passing through the surface is given in column $(\theta_1 - (\theta_0 - \varphi))$. The bending is observed to increase greatly as the angle φ that the surface makes with the reflecting planes increases. The values of δ obtained agree well with the theoretical values $\delta = 3.29 \times 10^{-6}$ and $\delta = 2.62 \times 10^{-6}$, respectively, as calculated by the Lorentz dispersion formula.

Theories of refraction indicate that the refraction should be much increased if the frequency of the incident radiation is near that of a resonance frequency of the refracting substance. This is well shown by the Lorentz form of the dispersion formula:

$$\delta = \frac{e^2}{2\pi m}\left(\frac{n_1}{\nu^2 - \nu^2{}_1} + \frac{n_2}{\nu^2 - \nu^2{}_2} + \text{etc.}\right),$$

where n_1, n_2, etc., are the number of electrons per unit volume having resonance frequencies ν_1, ν_2, etc.

This increased refractive effect was tested by reflecting CuK_α radiation from a properly cut crystal of pyrites.[6] The absorption (resonance) frequency of the k energy level of iron is sufficiently near the CuK_α frequency to permit a check on the formula. In this experiment it was sought to measure not only the refraction but the dispersion.

The pyrites crystal was ground at an angle as indicated in

[6] DAVIS and VON NARDROFF, *Proc. Nat. Acad. Sci.*, 10, 9 (1924). R. VON NARDROFF, *Phys. Rev.*, No. 2, Aug., 1924.

Fig. 1, and mounted on the spectrometer in the usual manner. Measurements were made at four incident frequencies, the two principal k radiations from molybdenum and copper. The k_β and k_α radiations of copper are sufficiently near the k resonance frequency of iron to produce an effect on the refraction.

In order to compare the experimental results with theory, the dispersion formula was written in the following form:

$$\delta = \frac{e^2N}{2\pi m}\left[\frac{K_{Fe}}{\nu^2 - \nu^2_{Fe}} + \frac{K_S}{\nu^2 - \nu^2_S} + \frac{(Z_{Fe} - K_{Fe}) + 2(Z_S - K_S)}{\nu^2}\right],$$

where N = numbers of FeS_2 molecules per c.c.

Z = atomic number.

K = number of electrons in k level.

The experimental results are given in the last column of Table II, while the calculated values of δ are given in the next to the last column.

TABLE II.

Radiation.	$\sin\theta_k$.	λ_{air}.	$\delta \times 10^6$ (calc.). $(k=0)$	$(k=1)$	$(k=2)$	$(k=3)$.	$\delta \times 10^6$ (obs.).
$MoK\alpha_1$	.1309235	.707717 ±15	3.29		3.31		3.35 ±.20
$MoK\beta_1$	.1167353	.63102 ± 6	2.62		2.64		2.87 ±.20
$CuK\alpha_1$	.2843818	1.53722 ±15	15.58	16.58	17.60	18.61	17.6 ±.5
$CuK\beta_1$	.2569866	1.38915 ±15	12.69	13.12	13.53	13.95	13.2 ±.4

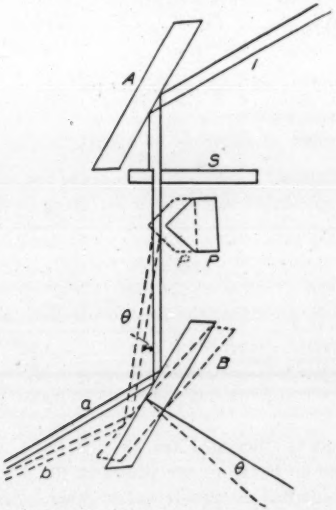
The formula is calculated for $k = 0, 1, 2, 3$. The meaning of $k = 0$ is that no electrons are placed in the k level, but these electrons are regarded as transferred to other outer levels of the atom where their natural frequencies have no resonance response.

Inspection of the calculated values of δ for copper radiation shows that the agreement with experiment is much the best for $k = 2$. This directly confirms the Bohr model of the atom which assigns two electrons in the k level. Measurements of refraction thus furnish an independent method of counting the electrons in the energy levels. This method might be used to estimate the distribution of the eight L electrons in the three L levels. The experimental difficulties, however, would be great on account of the great absorption of X-rays by elements of high atomic number.

REFRACTION BY PRISMS.

Since X-rays are really refracted it should be possible by sufficient refinement of apparatus to measure the bending on passing through a prism as in the case of light. This has been accomplished in the laboratory of Professor Siegbahn.[7] The refractive effect is made as large as possible by using a prism of very large

FIG. 2.



The double X-ray spectrometer.

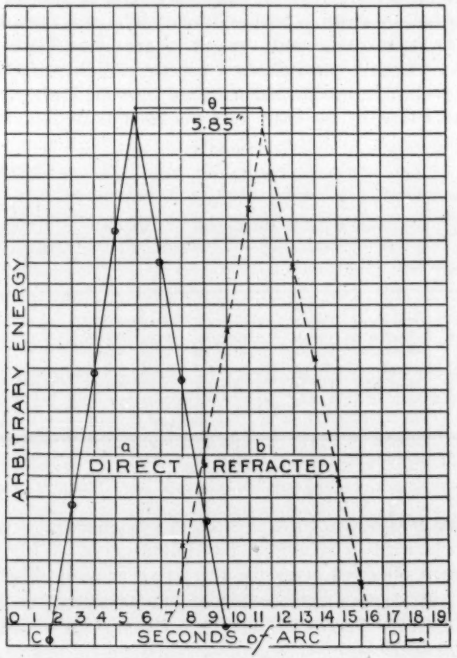
angle. The beam of X-rays passes over and through the extreme obtuse point of the prism, and then falls on a photographic plate. One part of the beam is refracted and another part is totally reflected. The results obtained by this method confirm those by the cut crystal method but are possibly not quite so accurate. The method is subject to limitation that the refracting edge must be ground with great accuracy.

A method of measuring the minute bending of the rays on passing through a prism has been devised by the writer. The

[7] LARRSON, SIEGBAHN and WALLER, *Naturious*, **52**, 1212 (1924).

double X-ray spectrometer which has been used so extensively for the measurement of crystal reflectivity is especially suited to the measurement of refraction in a prism.[8] The arrangement of

FIG. 3.



Specimen curves showing the refraction in an aluminum prism.

the two crystals, crystal *A* the collimater and crystal *B* the analyzer, and the prism are shown in Fig. 2. The method is at once simple and capable of considerable accuracy.

The beam of homogeneous X-rays is reflected in a parallel beam from *A* and falls on *B*. When surface of crystal *B* is

[8] DAVIS, BERGEN, and SLACK, *Phys. Rev.*, Jan., 1926. C. M. SLACK, *Phys. Rev.*, June, 1926.

parallel to that of crystal A maximum reflection is obtained. If crystal B is now rocked through a very small angle each side of this position very sharp curves are obtained which are known as rocking curves.

These curves are very narrow. We have obtained curves not more than 6″ in width at half maxima. The peaks of such curves may be located to one-tenth second of arc by sufficient care and repetition. Such a curve is shown in the full line of Fig. 3.

The prism is now pushed forward into the path of the beam by a fine screw. The beam is bent as indicated in the figure.

TABLE III.

Data on Refraction of X-rays. Values of $\delta (= 1 - \mu)$.

Substance.	Prism Angle (degrees).		Refraction (sec. of arc).	$\delta \times 10^6$ observed.	$\delta \times 10^6$ Calc. by Lorentz Formula.	
					Complete.	Omitting Char. Freq.
			.7078 A.			
Silver.....	single	63.5	2.42 ±0.1	5.85 ±0.3	5.80	6.33
Copper.....	single	60.0	2.12 ±0.2	5.95 ±0.5	5.72	5.60
Sulphur.....	double	168.3	5.39 ±0.2	1.39 ±0.05	1.37	1.37
Aluminum...	double	166.0	5.62 ±0.2	1.68 ±0.07	1.77	1.77
Carbon.....	single	86.4	4.02 ±0.5	1.23 ±0.15	1.10	1.10
Celluloid....	double	171.0	5.12 ±0.1	.980 ±0.03	.991	.991
Paraffin.....	double	173.0	4.71 ±0.15	.701 ±0.03	.714	.714
			$\lambda = 1.537$ A.			
Aluminum...	double	116.0	5.53 ±0.1	8.40 ±0.2	8.46	8.44
Celluloid....	double	144.0	6.06 ±0.1	4.78 ±0.1	4.71	4.71
Paraffin.....	double	152.0	5.40 ±0.15	3.28 ±0.15	3.37	3.37

The crystal B is now turned to catch this beam and a new rocking curve is obtained as shown in the dotted curve of Fig. 3. The displacement obtained gives the refractive bending directly without any correction. This particular experiment was made with an aluminum prism whose angle was 166°. The radiation was MoK_α ($\lambda = .7078$).

The complete results obtained for a number of prisms of various angles and substances are shown in Table III. The calculated value of $\delta = 1 - \mu$ is given in the last columns of the table. The theory of refraction is quite completely confirmed.

The dispersion formula shows that if the applied radiation has a frequency ν very near a resonance frequency ν_1, the term containing these frequencies becomes very large. As ν crosses the value ν_1 the dispersion curve for this term has two branches

extending to infinity at $\nu = \nu_1$. However, this is not true if the vibrating electron is subject to dissipative forces which must be the case. If dissipative forces are present, the two branches do not go to infinity but pass through zero.

Attempts to measure the refraction of X-rays near the absorption limit are attended with great difficulties owing to the great absorption of the rays, and the difficulty of producing intense radiation of the proper frequencies.

Experiments by Davis and Slack [9] indicate that probably the curves neither go to infinity nor do they pass through zero at $\nu = \nu_1$, as is the case in the dispersion of sodium light in sodium vapor.

The X-ray resonance at an absorption limit, however, is not symmetrical. It is sharp on one side and diffuse on the other. An investigation of the theory of dispersion for this unsymmetrical resonance by R. de L. Kronig [10] indicates that the dispersion curves should not be symmetrical on each side of zero and should not pass through zero. The experiments of Davis and Slack so far as they go confirm the results of Kronig.

The bending of the X-rays on passing through a prism are in the opposite direction to the bending of a beam of light. That is since $\mu = 1 - \delta$ and δ is positive, μ is less than unity and the *apparent* velocity is greater than that of light in free space. This is an interesting case of the difference between wave velocity and group velocity. This increase of the *apparent* velocity is to be expected in the case where the frequency of the radiation is greater than that of the greater part of the electrons of the matter which the rays traverse.

MEASUREMENT OF THE SIZES OF FINE PARTICLES.

An interesting phenomenon was observed while measuring the refraction in prisms with the double X-ray spectrometer. The rocking curves which are very narrow when the rays pass through a homogeneous substance were found to be much widened when the prism was made of a granular substance. This effect has been investigated by Slack.[11] The rays were passed through slabs of graphite of various thicknesses. The broadening observed is

[9] DAVIS and SLACK, *Phys. Rev.*, **25**, 881 (1925).
[10] KRONIG, *Jour. Am. Opt. Soc.*, June, 1926.
[11] SLACK, *Phys. Rev.*, June, 1926.

proportional to the thickness of the slab traversed by the rays and proportional to the square of the wave-length of the X-rays. This immediately suggests that the effect is due to refraction of the rays at the surfaces of the fine particles composing the graphite.

TABLE IV.

Data on the Broadening of Refraction Curves in Graphite.

Thickness of Graphite.	Curve Width at Half Max.	Broadening.	
	MoKα.		
13.8 mm.	58 sec.	52 sec.	±5
8.4	46	40	±5
4.3	30	24	±3
2.2	19	13	±3
1.2	15	9	±2
.62	10	4	±2
0.	6	0	...
	CuKα.		
1.2	58	47	±5
.62	45	34	±4
0.	10.8	0	...

The theory of this broadening due to refraction has been worked out by Von Nardroff [12] for the case of a large number of minute spheres having a random distribution.

The radius R of the particles assumed to be spheres is expressed as follows:

$$R = \frac{6\upsilon D\delta^2 \log\left(\frac{2}{\delta} + 1\right)}{(w^2_1 - w^2)},$$

where D = thickness that the rays pass through.

υ = actual volume of spheres per c.c.

$\delta = 1 - \mu$.

ω = width of rocking curve (normal).

ω_1 = width of rocking curves after passing through the powder.

The validity of this equation for the dimensions of fine particles was tested by means of fine aluminum powder of known dimensions. The aluminum powder was passed through fine

[12] R. VON NARDROFF, *Phys. Rev.*, **28**, No. 2 (1926).

sieves and then a large number of them individually measured with a microscope. They were in general not spherical in shape but irregularly ellipsoidal. The dimensions major and minor ranged from .005 cm. to .015 cm. Measurements on the spectrometer with a slab composed of these fine particles broadened the rocking curves considerably. The diameter of the particles calculated from this broadening by means of the equation gave a value of .007 cm. mean. This is a fairly good check considering that the particles were irregular in shape.

This method should make it possible to measure the size of small particles far beyond the range of microscopic vision. It might be possible to measure particles as small as 10^{-7} cm. in diameter. For this purpose the particles must be composed of a substance whose density and whose refractivity for X-rays may be measured in the solid form.

Action of Hydrogen upon a Mixture of Cuprous Oxide and Zinc Oxide.—According to WILLIAM ROGERS, JR., of Temple University (*Jour. Am. Chem. Soc.*, 1927, **49**, 1432–1435), when hydrogen is passed over a mixture of cuprous oxide and zinc oxide at a temperature of 300° C., the zinc oxide is completely reduced to the metal. J. S. H.

Fixed Nitrogen and the Fertilizer Industry. (U. S. Department of Agriculture Press Service.)—The United States, which five years ago had no plants for the fixation of atmospheric nitrogen, now has seven synthetic ammonia installations with a combined capacity of about eighty tons a day. While none of this output is finding its way as yet into agricultural use, it is having the indirect effect of forcing additional quantities of by-product ammonia into the fertilizer market. Growth of the nitrogen-fixation industry in the United States is certain eventually to be an important factor in promoting soil fertility.

Projected plans for the expansion of the industry indicate that before many years the products of atmospheric fixation will be competing directly in this country with other sources of fertilizer nitrogen. In the synthetic-ammonia process, the method of atmospheric nitrogen fixation in which practically all commercial effort in this country is concentrated, purified hydrogen and nitrogen gas are made to combine at high pressures and temperatures so as to form ammonia. The ammonia thus obtained is readily transformed into salts suitable for use as fertilizer.

Fixed nitrogen, because of its limited occurrence in nature, presents the greatest problem in the maintenance of soil fertility. Nitrogen from the air is the logical ultimate source of supply, since free nitrogen comprises four-fifths of the air. The development of atmospheric nitrogen fixation promises greatly to increase the world's supply of nitrogen, and incidentally to promote improvement in the production of fertilizer materials. Already better and more concentrated fertilizer salts are being produced and marketed as a result of the competition of synthetic-nitrogen plants.

Although Germany, with about 70 per cent. of the world's production of atmospheric nitrogen, is still the centre of the nitrogen-fixation industry, expansion in other countries has been very rapid in recent years. Atmospheric nitrogen has very largely supplied the increased demand in recent years for inorganic nitrogen. There are three principal sources of combined nitrogen, namely, atmospheric fixation, Chilean nitrate, and by-product ammonia. The progress of the nitrogen-fixation industry is indicated by the fact that in 1925 the world's production of nitrogen by atmospheric fixation was 607,000 metric tons, compared with a production of 340,000 metric tons of Chilean nitrate, and 330,000 metric tons of by-product ammonia. Probably nearly 90 per cent. of this total of 1,277,000 metric tons was used in agriculture.

THE CONTRAST OF PHOTOGRAPHIC PRINTING PAPER.*†

BY

LOYD A. JONES.

Research Laboratory, Eastman Kodak Company.

SECTION E. CORRELATION BETWEEN SENSITOMETRIC CONSTANTS OF POSITIVE MATERIALS AND THE CHARACTERISTICS OF THE OPTIMAL POSITIVES AND NEGATIVES.

A CASUAL inspection of the data relating to characteristics of the *positive materials*, the *optimal positives*, and the *optimal negatives* indicates the existence of certain more or less definite quantitative relationships. It seems worth while now to analyze these relationships more critically with the hope of acquiring a more thorough understanding of how to obtain the best possible photographic quality by choosing from a wide variety of developing-out papers available the one best adapted for the rendition of a particular negative. The general principles controlling the choice of a positive material to fit a given negative are quite generally recognized and understood in a qualitative way. For instance, the practical worker knows that a soft negative, one which has been developed for a relatively short time or in which the brightness scale of the object reproduced is short, must be printed on a contrasty or hard paper (that is one having a short exposure scale) in order to obtain a positive of satisfactory quality. On the other hand, if the negative is hard, due either to long development or to great brightness contrast in the object, the paper must possess a great exposure scale in order to yield a satisfactory result. It is apparent, therefore, that a more or less definite relation exists between the density scale of the negative and the exposure scale of the positive material yielding a satisfactory reproduction of the negative. An inspection of the data presented in the previous section of this communication indi-

* Communicated by Dr. C. E. K. Mees, Director of Laboratory, and published as Communication No. 264 from the Research Laboratory of the Eastman Kodak Company.

† Continued from p. 156 of the January issue.

cates that there is not, however, a precise quantitative relationship between these two factors. As was pointed out in the previous section, the proportion of the total exposure scale of the positive material which is available for the rendition of the negative depends very much upon other factors. A careful study of the available data has shown some very definite systematic relationships between the various factors involved and in this section these relationships will be discussed in detail.

I. GENERAL REVIEW OF TONE REPRODUCTION THEORY WITH APPLICATION TO A SPECIFIC CASE.

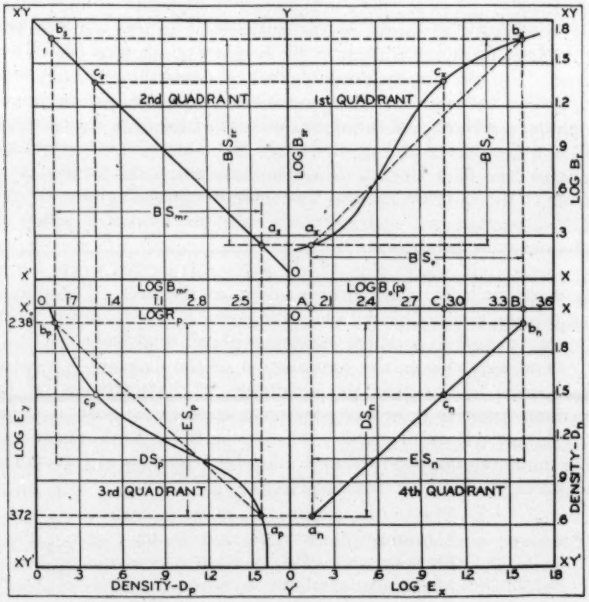
A rather thorough knowledge of the principles of tone reproduction is required for understanding the significance of the results derived in this section. It may be well, therefore, to review briefly some of the principles of tone reproduction and to explain in greater detail the construction and interpretation of the reproduction diagram which was shown in Fig. 4, Section A (p. 199). The diagram mentioned was drawn to represent a perfectly general case using hypothetical characteristic curves for the positive and negative materials. This will now be applied to a specific case in order to show the character of the tone reproduction obtainable on a positive material with a negative of optimal quality. For this purpose positive material No. 9 (Azo F hard) has been chosen. A complete diagram, showing the tone reproduction obtained on this material when used with the negative which was found by the statistical method to be of optimal quality, is shown in Fig. 52. On the line OX a suitable scale of object brightness, log $B_0(p)$, is established. Referring to Section B, Table VIII (p. 483), it will be found that the following values were determined relative to the object:

Maximum photographic brightness, $B_0(p) = 3.40$.
Minimum photographic brightness, $B_0(p) = 1.95$.
Photographic brightness scale of object, $BS_0(p) = 1.45$.
Maximum visual brightness, $B_0(v) = 3.40$.
Minimum visual brightness, $B_0(v) = 1.81$.
Visual brightness scale of object, $BS_0(v) = 1.59$.

It will be noted that the brightness scale as measured visually is somewhat greater than when measured photographically. This is due either to lens flare or to the colors present in the object photographed. The lowest brightness found in the object was

the trunk of the willow tree, which is a dark brownish-gray in color. The highest brightness measured was the awning in sunlight. This awning was made of white and yellow striped canvas. The hue of a dark brown is the same as that of a yellow, the difference in appearance being due largely to luminosity. The

FIG. 52.



Tone reproduction diagram applied to the optimal positive and negative for positive material No. 9.

highest and lowest brightness measurements therefore were made on colors of approximately the same hue. Since a panchromatic plate was used in making the negatives, it does not seem possible that the observed difference between the photographic and visual brightness scales can be attributed to color differences. It is highly probable that the lower value of brightness scale, as measured photographically, is due to the presence of lens flare, which

consists of light reflected from the various glass air surfaces of the lens system and distributed fairly uniformly over the surface of the negative during exposure. The object, as seen by the photographic plate, therefore, has a somewhat lower brightness scale than the object as seen by the eye. In the final evaluation of tone reproduction which consists of a comparison of the visually observed brightness values in the object with the brightness values of corresponding areas in the positive, it is necessary to take into account this distortion of the brightness relations due to lens flare. For the present purpose, that of establishing definite relationships between the characteristics of the optimal negatives, optimal positives, and the characteristics of the tone reproduction, it is quite permissible to disregard the distortion of brightness due to lens flare since it is a common factor and present in all cases. In constructing the tone reproduction diagram for this particular case we shall therefore use the measured values of photographic brightness.

Referring now to Fig. 52, the point A is located where log B_0 is equal to 1.95, this corresponding to the area of minimum brightness in the object; while the point B is established where log B_0 is equal to 3.40, the maximum object brightness.

The next step in the construction of the diagram is to place the curve representing the sensitometric characteristics of the optimal negative in proper position relative to the scale of object brightness. From Table XVIII, Section B (p. 513), the following data are obtained relating to the characteristics of the optimal negative for positive material No. 9.

Maximum density, D_n max. = 2.00.
Minimum density, D_n min. = .67.
Density scale, DS_n = 1.33.
Maximum gradient, γ_n = .94.
Average gradient, ϕ_n = .94.

On the line $X - XY'$ a density scale is established on which the maximum and minimum values of negative density shown above may be plotted. On the ordinate passing through point B at a density value of 2.00 the point b_n is located, while on the ordinate passing through the point A at a density value of .67 the point a_n is located. Since ϕ_n is equal to γ_n, it follows that the entire portion of the negative material characteristic used in making this optimal negative is a straight line. Hence a straight line drawn

between the points a_n and b_n represents the density-log E characteristic of the optimal negative. Since in this problem we are not interested particularly in absolute values of exposure, the scale of log exposure established on the line $Y' - XY'$, as shown in the figure, is only relative.

The characteristic curve of the positive material is now plotted in the third quadrant of the diagram in correct relationship to the characteristic curve of the optimal negative. Referring to Table XVIII of Section B (p. 513), the following data relative to the optimal positive (No. 9) are found:

Maximum density, D_p max. = 1.57.
Minimum density, D_p min. = .13.
Density scale, DS_p = 1.44.
Exposure scale, ES_p = 1.34.
Average gradient, ϕ_p = 1.07.

Along the line $X'Y' - Y'$ is established a suitable scale of density. On a horizontal line passing through a point a_n the point a_p is located where D_p is equal to 1.57, the maximum density in the optimal positive. On a horizontal line passing through the point b_n, the point b_p is established where D_p is equal to .13, the minimum density in the optimal positive. In this way two points of the characteristic curve for material No. 9 are established. The complete characteristic curves for all the materials used are available, having been determined in Section C, which deals with the sensitometric phase of the subject. The complete characteristic curve for this material is shown in Fig. 36, Section D. From this curve it is possible to determine the log exposure values corresponding to the points D_p = .13 and 1.57. This establishes the log exposure scale on the line $X' - X'Y''$. It is now possible to plot the complete characteristic curve as shown. On the line OX' a scale of reflecting power, log R_p, is established. The numerical values of this scale are related to those on the scale of positive density, D_p, by the equation

$$D_p = \log \frac{1}{R_p} \cdot \qquad (1)$$

On the line $O'X'$ is established a suitable scale for designating the brightness of the various print areas. The numerical values on this scale are related to those on the scale of log R_p directly below it by a constant factor which is dependent upon the illu-

mination incident upon the positive. Thus if this illumination be represented by I_p then, for corresponding points on the two scales, the following relationship must exist:

$$\log R_p + \log I_p = \log B_{mr}. \qquad (2)$$

These brightnesses are referred to as brightnesses of the *material reproduction*. This term "material reproduction" is used in referring to the purely physical aspects of the brightness distribution in the positive. The reproduction as seen by the eye under any conditions of illumination is referred to as the *subjective reproduction* and factors relating thereto are designated by the subscript *sr*. It is very important in the complete theory of tone reproduction to distinguish carefully between the characteristics of the *material reproduction* and the *subjective reproduction*.

The second quadrant is utilized for introducing a curve by means of which the characteristics of the subjective reproduction may be obtained from the characteristics of the material reproduction. The shape and position of this curve can be computed from known relationships between the brightness discrimination function for the eye, as adapted to the illumination levels under which the object is viewed, and the same function applying to the eye, as adapted to the brightness levels existing when the positive is observed. In the case of the particular problem with which we are dealing at this time, it is permissible to assume that this curve is satisfactorily represented by a straight line passing through the origin at an angle of 45° to the log B_{mr} axis. This follows from the fact that the illumination level under which the judgments of photographic quality were made was intentionally adjusted to a sufficiently high level so that the eyes of the observers were operating on the flat portion of the brightness discrimination curve as was also the case when the original object was being observed under the existent illumination levels. Under these conditions any interval on the B_{mr} scale is represented by an interval of corresponding magnitude on the BS_{sr} scale. Thus the total brightness scale of the material reproduction is equivalent in magnitude to the brilliance scale of the subjective reproduction and under these limiting conditions it is unnecessary to distinguish between the subjective and material reproduction characteristics. Throughout the remainder of the treatment of this subject, therefore, factors relating to the reproduction will be designated simply by the subscript *r*.

The point a_s is located at the intersection of a perpendicular through the point a_p with the line $O - X'Y$, while the point b_s is located where a perpendicular through the point b_p cuts the line $O - X'Y$. Horizontal lines drawn through the points a_s and b_s locate on the log B_{sr} scale (constructed on the line $O'Y$) the points corresponding to the minimum and maximum object brightnesses. Finally the point a_x is established at the intersection of a perpendicular through the point A with a horizontal line through the point a_s, and the point b_x is located at the intersection of a perpendicular through the point B with a horizontal line through the point b_s. The points a_x and b_x represent the extremities of the reproduction curve and points intermediate between these two are established in an exactly similar manner. For instance, from any point C on the log $B_0(p)$ scale a perpendicular line cuts the curve of the negative locating point c_n. A horizontal line through this establishes the point c_p on the curve of the positive material. A perpendicular through this point locates the point c_s and the horizontal line through the point c_s intersecting a perpendicular through C establishes the point c_x. In this way a complete curve showing the relation between the brightness of various areas in the positive as a function of the brightness distribution in the object is obtained. This is termed the *reproduction curve* and its shape and position give quantitative information as to the precision with which the object brightnesses have been reproduced. Since the interpretation of the curve has been discussed in the previous publication, it will be unnecessary to treat this subject at present. Contrast and photographic quality are intimately related to gradient characteristics, hence we are chiefly interested at present in the relations between the gradient characteristics of the negative, positive, and reproduction curves. The symbol G is used in general to represent gradient, the slope at any point on a curve given by the differential dy/dx.

G_n = gradient of negative characteristic;
$= dD_n/d \log E_x$.

G_p = gradient of the positive characteristic;
$= dD_p/d \log E_y$.

G_r = gradient of the reproduction characteristic;
$= d \log B_r/d \log B_0$.

It has been shown previously [24] that for corresponding points,

such as c_n, c_p, and c_x, on the various curves the following relations between the gradients exist:

$$G_n \cdot G_p = G_r . \tag{3}$$

For the straight-line portions of these characteristics,

$$\gamma_n \cdot \gamma_p = \gamma_r . \tag{4}$$

It is evident that the same form must hold for average gradients and we may therefore write

$$\phi_n \cdot \phi_p = \phi_r . \tag{5}$$

Since the above relationships between gradients on the three characteristics at corresponding points are perfectly general, it follows that they will apply at the limiting points, such as a_n and b_n.

Therefore

$$h\theta_n \cdot h\theta_p = h\theta_r , \tag{6}$$

$$s\theta_n \cdot s\theta_p = s\theta_r . \tag{7}$$

It is evident from the geometrical construction of the tone reproduction diagram that:

$$BS_0 (p) = ES_n \tag{8}$$

$$DS_n = ES_p \tag{9}$$

$$DS_p = BS_r . \tag{10}$$

Some other relationships applying to this tone reproduction diagram and having important bearing on the subject of contrast, the correlation of negative and positive characteristics, will be treated later in this paper.

II. SUMMARY OF DATA AVAILABLE FOR USE IN CORRELATION.

The data relating to the characteristics of the *positive materials*, the *optimal negatives* and *positives*, and the *object* photographed are scattered through the four preceding sections. For convenience the values which are to be used in this section on correlation have been retabulated in Tables XXXVII and XXXVIII so that they can be readily referred to. In Table XXXVII are given the data relative to the positive materials themselves. These values are all of *sensitometric* origin. They are measured directly by physical methods and do not depend in any way upon judgment of photographic quality. They are of

TABLE XXXVII.

Sensitometric data relative to positive materials.

	D_y (max.).	DS_y.	ES_y.	ϕ_y.	DL_y.	γ_y.	τ_y.	Ω_y.
1	1.15	1.07	1.85	.58	.52	.83	.67	.72
2	1.19	1.12	1.74	.64	.58	.94	.77	.86
3	1.22	1.17	1.20	.97	.56	2.07	1.44	1.69
4	1.40	1.34	1.79	.75	.87	1.13	.96	1.28
5	1.40	1.36	1.69	.80	.55	1.37	1.01	1.37
6	1.48	1.42	1.43	.99	.71	2.29	1.54	2.20
7	1.41	1.37	1.29	1.06	.63	2.62	1.85	2.53
8	1.52	1.47	1.87	.83	.75	1.34	1.07	1.58
9	1.60	1.52	1.61	.95	.73	1.87	1.33	2.03
10	1.67	1.61	1.54	1.05	.83	2.37	1.73	2.79
11	1.70	1.67	1.36	1.23	.75	3.12	2.08	3.46
12	1.66	1.61	1.57	1.03	.74	1.85	1.30	2.10
13	1.70	1.65	1.44	1.15	.74	3.08	1.98	3.28
14	1.23	1.18	1.51	.78	.49	1.40	1.08	1.27
15	1.21	1.17	1.32	.89	.55	1.67	1.23	1.44
16	1.50	1.46	1.71	.85	.53	1.89	1.34	1.92
17	1.40	1.37	1.24	1.10	.43	2.26	1.52	2.08
18	1.55	1.51	1.38	1.09	.90	2.25	1.74	2.63
19	1.55	1.52	1.12	1.36	.79	2.63	1.96	2.98
20	1.16	1.12	1.45	.77	.65	1.18	.98	1.09
21	1.52	1.49	.93	1.60	.90	3.21	2.48	3.69
22	1.50	1.45	1.59	.91	.95	1.23	1.03	1.49
23	1.21	1.15	1.65	.70	.73	.94	.83	.95
24	1.22	1.17	1.68	.70	.55	.92	.74	.89
25	1.23	1.17	1.60	.73	.53	1.13	.88	1.03
26	1.14	1.10	1.61	.68	.53	.98	.79	.87
27	1.47	1.40	1.44	.97	.81	1.37	1.11	1.56
28	1.52	1.46	1.56	.94	.65	1.66	1.25	1.82
29	1.55	1.49	1.33	1.12	.76	2.53	1.80	2.68
30	1.28	1.20	1.85	.65	.54	1.01	.96	1.15
31	1.36	1.30	1.74	.75	.55	1.17	.91	1.18
32	1.40	1.33	1.68	.79	.76	1.22	.99	1.32
33	1.16	1.09	1.72	.63	.71	.86	.76	.83
34	1.52	1.46	1.73	.84	.80	1.19	.99	1.45
35	1.28	1.20	1.83	.66	.58	1.14	.92	1.10
36	1.57	1.49	1.61	.93	.75	1.41	1.10	1.64
37	1.19	1.12	1.66	.67	.61	.98	.79	.88

TABLE XXXVIII.

Statistical data relative to the optimal negatives, optimal positives, and the photographic reproduction curve. While these values are sensitometric in form they are directly dependent upon the statistically determined gradient of the negative which yields a positive of best possible photographic quality on each of the positive materials.

	Optimal Negatives.			Optimal Positives.					Photographic Reproduction.		
	DS_n.	γ_n.	ϕ_n.	$e\theta_p$.	DS_p.	ϕ_p.	τ_p.	Ω_p.	γ_r.	ϕ_r.	$e\theta_r$.
1	1.80	1.34	1.28	.21	1.06	.59	.68	.72	1.11	.76	.27
2	1.52	1.12	1.10	.27	1.05	.69	.80	.84	1.05	.76	.30
3	.83	.57	.57	.48	1.05	1.26	1.56	1.64	1.18	.72	.27
4	1.61	1.18	1.15	.27	1.30	.81	.97	1.26	1.30	.93	.31
5	1.39	1.00	.99	.34	1.27	.91	1.05	1.33	1.36	.90	.34
6	1.00	.71	.71	.47	1.30	1.30	1.65	2.15	1.62	.92	.33
7	.81	.54	.54	.55	1.18	1.46	2.07	2.44	1.41	.79	.30
8	1.70	1.24	1.20	.30	1.43	.84	1.08	1.55	1.61	1.01	.36
9	1.34	.94	.94	.40	1.42	1.06	1.38	1.96	1.75	1.00	.38
10	.96	.66	.66	.58	1.43	1.49	1.88	2.69	1.56	.98	.38
11	.71	.46	.46	.75	1.39	1.96	2.40	3.33	1.43	.90	.35
12	1.16	.82	.82	.46	1.46	1.22	1.38	2.02	1.52	1.00	.38
13	.70	.46	.46	.76	1.32	1.89	2.36	3.12	1.42	.87	.35
14	1.31	.92	.92	.33	1.13	.86	1.11	1.25	1.26	.79	.30
15	.95	.64	.64	.46	1.04	1.10	1.33	1.39	1.07	.70	.29
16	1.29	.91	.91	.41	1.36	1.05	1.38	1.88	1.72	.96	.37
17	.89	.60	.60	.58	1.24	1.40	1.63	2.02	1.36	.84	.35
18	1.01	.72	.72	.46	1.41	1.40	1.83	2.59	1.62	1.01	.33
19	.72	.47	.47	.74	1.33	1.85	2.17	2.89	1.24	.87	.35
20	1.31	.96	.96	.28	1.09	.83	1.00	1.09	1.13	.80	.27
21	.59	.40	.40	.78	1.33	2.26	2.71	3.60	1.28	.90	.31
22	1.39	1.01	1.00	.35	1.39	1.00	1.05	1.47	1.23	1.00	.35
23	1.51	1.11	1.09	.27	1.13	.75	.82	.93	1.03	.82	.29
24	1.54	1.11	1.09	.27	1.14	.74	.75	.86	1.00	.81	.29
25	1.41	1.02	1.01	.29	1.13	.80	.89	1.01	1.14	.81	.29
26	1.52	1.10	1.08	.25	1.09	.72	.80	.87	1.06	.78	.27
27	1.22	.83	.83	.37	1.32	1.08	1.15	1.52	1.14	.90	.31
28	1.15	.79	.79	.46	1.32	1.15	1.33	1.76	1.31	.91	.36
29	.82	.54	.54	.59	1.30	1.59	2.00	2.60	1.37	.86	.32
30	1.49	1.07	1.05	.30	1.13	.76	1.00	1.13	1.06	.80	.32
31	1.43	1.03	1.02	.34	1.22	.85	.94	1.15	1.19	.87	.35
32	1.40	1.01	1.00	.33	1.26	.90	1.03	1.30	1.22	.90	.33
33	1.68	1.25	1.21	.22	1.09	.64	.76	.83	1.04	.77	.27
34	1.50	1.09	1.07	.32	1.40	.93	1.01	1.42	1.27	1.00	.34
35	1.71	1.24	1.20	.24	1.18	.69	.92	1.08	1.37	.83	.29
36	1.32	.93	.93	.38	1.39	1.05	1.15	1.60	1.30	.98	.35
37	1.51	1.11	1.09	.26	1.08	.72	.81	.87	1.07	.79	.28

course subject to the usual errors of measurement as outlined in the first part of the section on sensitometric results (Section C).

The tabulated values are as follows:

D_y max. = maximum density
DS_y = density scale
ES_y = exposure scale
ϕ_y = mean gradient of the characteristic curve
DL_y = density latitude
γ_y = the slope of the straight-line portion of the characteristic curve.

All of the above values are read directly from the characteristic curve and do not involve any computation.

τ_y = the rate factor of contrast,
Ω_y = contrast.

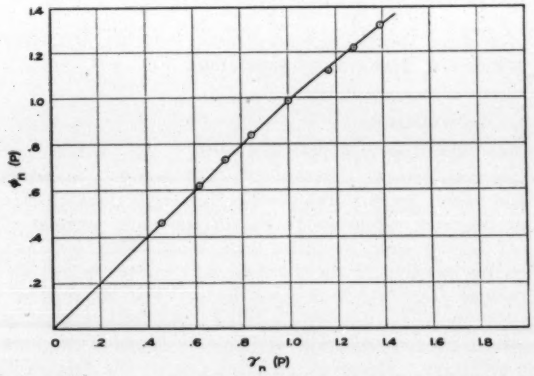
These two values are computed according to the method outlined in the previous section (Section D). It should be remembered that these values apply to the positive materials themselves without any reference whatever to the prints made thereon. The values of density scale, exposure scale, mean slope, and contrast apply to the portion of the characteristic curve limited by the points where $dD/d \log E$ is equal to .2. The assumption that portions of the curve for which $dD/d \log E$ is less than .2 are of no use in the reproduction of negative density differences is entirely arbitrary. However, an inspection of the data in Table XXXII of the last section shows that this assumption is justified since it was not found possible to obtain a print of satisfactory quality in the case of any of the positive materials used by using a negative having a density scale so great as to require utilization of portions of the positive material curve having gradient values of less than .2.

In Table XXXVIII are tabulated the values relating to the characteristics of the optimal negatives and positives. These values are all of *statistical* origin and relate to the characteristics of the negatives, which, according to the average judgment of seventy-five observers, gave the best possible photographic quality when used with the various positive materials; and to the characteristics of the best possible positives which can be made on the various positive materials by use of the optimal negatives.

The values of density scale and γ for the optimal negatives

are taken directly from the final table of Section B (Table XVIII). The values of mean gradient (ϕ_n) for these optimal negatives are read from the curve shown in Fig. 53. This curve shows $\phi_n(p)$ as a function of $\gamma_n(p)$ and is plotted by using the data in the last two columns of Table VI, Section B. In making a final analysis of the quality of tone reproduction obtained, it is desirable to determine values of mean gradient for the optimal positives. As pointed out previously, this mean gradient is equal

FIG. 53.



Relation between maximum gradient, γ_n, and mean gradient, ϕ_n, for the test negatives.

to the product of the mean gradient of the negative by the mean gradient of the portion of the characteristic curve of the positive material used in making the print. Now reference to Fig. 8 in Section B will show that in case of negatives Nos. 1 to 6, inclusive, the entire scale of object brightness was rendered on the straight-line portion of the negative materials, hence in these cases ϕ_n is identical in value to γ_n. For negatives Nos. 7 and 8, however, the highest brightnesses were rendered on a portion of the curve departing somewhat from the straight line. In these cases ϕ_n is somewhat less than γ_n. The values of ϕ_n shown in Table XXXVIII are therefore used in computing the values of mean gradient of the reproduction curve. As a matter of fact, these values are also used in computing values of *limiting gradient* for the reproduction curve. The author realizes that a slight inaccuracy is introduced by this procedure since the limiting gradients

at the high density end of negatives Nos. 7 and 8 are slightly less than the values of ϕ_n for these negatives. However, the difference between γ_n and ϕ_n is less than 5 per cent. in the case of all optimal negatives except that for positive material No. 1. The error introduced by the use of the value of ϕ_n is considered so slight as to be of negligible importance.

The values, shown in the second section of Table XXXVIII, applying to the optimal positives, are

$e\theta_p$ = equalized limiting gradient
DS_p = density scale
θ_p = mean gradient
τ_p = rate factor of contrast
Ω_p = contrast.

Of these $e\theta_p$, DS_p, and θ_p are taken from Table XXXII in Section D. It should be remembered that these values apply to the optimal positives for which the printing exposure has been so adjusted to give a limiting gradient at the high-light end of the scale equal to the limiting gradient at the shadow end. The author wishes to emphasize the fact that this procedure in making the prints from the optimal negatives has been found to give in all cases the best possible photographic quality. The values of τ_p and Ω_p are taken from Table XXXIV, Section D, and apply to that portion of the characteristic curve of the material which was used in making the optimal positives. They therefore represent contrast and the rate factor of contrast for the portion of the curve which is useful in reproducing the particular object used in this work. It should also be remembered that the extent factor of contrast for the portion of the curve utilized is equal to the density scale of the positive.

In the third section of Table XXXVIII are given values relating to the photographic reproduction. These are computed according to the following equations:

γ_r = the gradient of the straight-line portion of the reproduction curve which is obviously also the *maximum gradient* (G_r max.).
$= \gamma_n \cdot \gamma_y$.
ϕ_r = average gradient of the reproduction curve.
$e\theta_r$ = equalized limiting gradient of the reproduction curve.
$= \phi_n \cdot e\theta_p$.
$= G_r$ min., minimum gradient of the reproduction curve.

It should be kept in mind that the values of gradient relating to the reproduction, as shown in Table XXXVIII, are based on the photographic value of the object brightness scale. As was pointed out previously, the brightness scale of the object as measured visually was somewhat greater than that measured photographically, this difference being attributable to the presence of lens flare. In any work where the ultimate object is the expression of the characteristics of the photographic reproduction in terms of the brightness scale as observed visually, this distortion due to lens flare cannot be neglected. From the standpoint of the mean gradient of the reproduction a satisfactory correction can be made by multiplying the values of mean gradient, ϕ_r, as tabulated in Table XXXVIII by the ratio of log $BS_0(p)$ to log $BS_0(v)$. In the particular case with which we are dealing this ratio is .91. Thus all of the mean gradient values as shown are approximately 10 per cent. higher than if this value were expressed in terms of the visual brightness scale of the object. The superposition of a veiling glare such as produced by light scattered from lens surfaces over the negative during exposure produces an actual distortion in the shape of the curve and hence when dealing with the distribution of brightness values lying between the upper and lower limits, it is not possible to use a correction factor of constant value. Since the correction in this particular case is relatively small, approximately 10 per cent., it is not considered necessary to make a precise correction for this factor. The same veiling glare was present in all cases and hence may be considered as a constant factor in the problem. We shall therefore confine our consideration of correlation between the statistical and sensitometric values to the relationships existing between the sensitometric and statistical values as based upon the photographic value of object brightness scale.

III. THE EXPRESSION OF EXPERIMENTAL ERRORS IN TERMS OF THE BRAVAIS-PEARSON COEFFICIENT.

The question of experimental errors involved in the measurement of sensitometric values was considered in detail in Section C, the conclusions drawn from this analysis being that these errors are of relatively low magnitude. The values shown in Table XXXVII are all of strictly *sensitometric* character and the probable error applying to these values can be computed directly from the errors inherent in the sensitometric measurements. The

values of Table XXXVIII are deduced by a series of graphical and numerical computations from those in Table XXXVII. It is evident, therefore, that an additional group of experimental uncertainties is involved. It must be remembered that the values of gamma for the *optimal negatives* (γ_n) were determined by a purely *statistical* method in which the probable errors involved are dependent upon an entirely different group of factors. However, after having assumed some particular value for γ_n in any case, the other numerical values shown in Table XXXVIII may be computed directly by numerical and graphical methods from the measured sensitometric characteristics of the positive materials, the optimal negatives, and the optimal positives.

It follows, therefore, that the various factors shown in Tables XXXVIII and XXXVII should be related directly to each other by certain tone reproduction equations. If any particular analytical relationship is assumed and the numerical values applying to the results for the different positive materials are inserted therein, it will be found that slight variations occur which may be attributed directly to experimental errors of various types quite apart from any probable error involved in the *statistical* determination of the gamma value (γ_n) of the *optimal* negative. The object of the work in this section is to determine, if possible, a relationship common to the entire group of the thirty-seven positive materials, between the sensitometric characteristics of the positive material and the negative which produces the optimal positive thereon. It is not reasonable to suppose that any such relationship which must necessarily depend essentially upon the statistically determined values of optimal negative gamma (γ_n) will hold precisely for all of the materials and it will be necessary to express in some way the perfection of correlation between the various factors.

Even if a perfect correlation, from the statistical standpoint, exists between two factors, the presence of the observational and computational uncertainties will prevent this appearing as a perfect correlation. It is necessary, therefore, to determine the lack of correlation which can be attributed directly to errors of a *purely experimental* and *computational* nature as distinct from the lack of correlation due to the *statistical* factors.

In the analysis of results obtained by statistical methods it is probable that the most satisfactory method of evaluating quanti-

tatively the correlation between two variables is in terms of the *Bravais-Pearson coefficient* or the *product-moment coefficient*, as it is sometimes called. By computing the correlation coefficient between two factors related theoretically in a known and definite manner and for which a large number of observed values are available, a value is obtained which may be taken as a measure of the total experimental errors involved. The correlation coefficient thus determined represents the maximum value which may reasonably be expected to exist for any statistical relationship based upon the same series of observations. In order to determine the lack of correlation which may be attributed to purely experimental errors, the values of DS_p, ϕ_n, and ϕ_p, as shown in Tables XXXVII and XXXVIII, have been used.

The theoretical relationship existing between these factors may be expressed by the following equations derived directly from the tone reproduction theory:

$$\phi_n \cdot \phi_p = \phi_r \tag{5}$$

$$\phi_r = ES_p/BS_0. \tag{11}$$

Therefore

$$DS_p = BS_0 (\phi_n \cdot \phi_p). \tag{12}$$

Since the brightness scale of the object (BS_0) is a constant, it is evident that equation (12) is of the straight-line form,

$$y = mx + k, \tag{13}$$

where $m = BS_0$

$x = \phi_n \cdot \phi_p$

and $k = 0$.

It is evident, therefore, if DS_p be plotted as a function of $(\phi_n \cdot \phi_p)$, that the resulting curve should be a straight line passing through the origin ($k = 0$) and having a gradient equal to the brightness scale of the object (BS_0). When the values of these factors as shown in Tables XXXVII and XXXVIII are plotted, it is found that all of the points do not lie on a straight line, thus indicating the presence of certain experimental errors.

The Bravais-Pearson correlation coefficient, r, is given by the following equation,

$$r = \frac{\Sigma ab}{\sqrt{(\Sigma a^2) \cdot (\Sigma b^2)}}. \tag{14}$$

The various factors appearing in this equation are determined from the following relationships:

$$\Sigma x = x_1 + x_2 \cdots + x_n$$
$$\Sigma y = y_1 + y_2 \cdots + y_n$$
$$\bar{x} = 1/n\, x$$
$$\bar{y} = 1/n\, y$$
$$a = x - \bar{x}$$
$$b = y - \bar{y}.$$

Since this same method will be used frequently in later parts of this paper, it may be well to illustrate in detail the method of computing the correlation coefficient from the data available. In the column designated as y, Table XXXIX, are entered the values of DS_p from Table XXXVIII, and in the column x are entered the values obtained by multiplying ϕ_n by ϕ_p, these values also being derived from Table XXXVIII. By addition of the numbers in these columns the summation values, Σy and Σx, shown at the bottom of the table, are obtained. These values divided by n, which in this case is 37, give the required values of $\bar{y}$ and $\bar{x}$. In the columns designated as b and a are the values of these factors as defined by the equations above. Squaring each of these gives the values shown in columns designated as b^2 and a^2, while in the last column of the table are the values obtained by multiplying b by a. At the bottom of the last three columns the summation values are shown. In this way the required values of Σab, Σa^2, and Σb^2 are obtained. Inserting these in equation (14), we have

$$r = \frac{.4083}{\sqrt{.2863 \times .6387}} = .955 = \text{correlation coefficient.}$$

For perfect correlation, that is all points lying on the same straight line, r should be equal to 1.0. It will be noted that the correlation coefficient obtained is very high, indicating that the accumulated experimental errors involved in all of the graphic and numerical computations are relatively small. A higher coefficient cannot be expected in the case of the correlation existing between factors which depend essentially upon statistical determinations. If it is found in the examination of relationships which depend upon the values determined statistically that the correlation coefficients approach fairly close to .95, it will be justifiable to conclude

TABLE XXXIX.

Data illustrating the method of computing the Bravais-Pearson correlation coefficient and the constants in the equation of the straight line for which Σh^2 is a minimum, h^2 being the shortest distance between the observed points and the straight line.

y	x	b	a	b^2	a^2	$b \cdot a$
1.06	.76	−.19	−.11	.0361	.0121	+.0209
1.05	.76	−.20	−.11	.0400	.0121	+.0220
1.05	.72	−.20	−.15	.0400	.0225	+.0300
1.30	.93	+.05	+.06	.0025	.0036	+.0030
1.27	.90	+.02	+.03	.0004	.0009	+.0006
1.30	.92	+.05	+.05	.0025	.0025	+.0025
1.18	.79	−.07	−.08	.0049	.0064	+.0056
1.43	1.01	+.18	+.14	.0324	.0196	+.0252
1.42	1.00	+.17	+.13	.0289	.0169	+.0221
1.43	.98	+.18	+.11	.0324	.0121	+.0198
1.39	.90	+.14	+.03	.0196	.0009	+.0042
1.46	1.00	+.21	+.13	.0441	.0169	+.0273
1.32	.87	+.07	+.00	.0049	.0000	+.0000
1.13	.79	−.12	−.08	.0144	.0064	+.0096
1.04	.70	−.21	−.17	.0441	.0289	+.0357
1.36	.96	+.11	+.09	.0121	.0081	+.0099
1.24	.84	−.01	−.03	.0001	.0009	+.0003
1.41	1.01	+.16	+.14	.0256	.0196	+.0224
1.33	.87	+.08	+.00	.0064	.0000	+.0008
1.09	.80	−.16	−.07	.0256	.0049	+.0112
1.33	.90	+.08	+.03	.0064	.0009	+.0024
1.39	1.00	+.14	+.13	.0196	.0169	+.0182
1.13	.82	−.12	−.05	.0144	.0025	+.0060
1.14	.81	−.11	−.06	.0121	.0036	+.0066
1.13	.81	−.12	−.06	.0144	.0036	+.0072
1.09	.78	−.16	−.09	.0256	.0081	+.0144
1.32	.90	+.07	+.03	.0049	.0009	+.0021
1.32	.91	+.07	+.04	.0049	.0016	+.0028
1.30	.86	+.05	−.01	.0025	.0001	−.0005
1.13	.80	−.12	−.07	.0144	.0049	+.0084
1.22	.87	−.03	+.00	.0009	.0000	−.0000
1.26	.90	+.01	+.03	.0001	.0009	+.0003
1.09	.77	−.16	−.10	.0256	.0100	+.0160
1.40	1.00	+.15	+.13	.0225	.0169	+.0195
1.18	.83	−.07	−.04	.0049	.0016	+.0028
1.39	.98	+.14	+.11	.0196	.0121	+.0154
1.08	.79	−.17	−.08	.0289	.0064	+.0136
46.16	32.24			.6387	.2863	+.4083

that the correlation between these values is of as high an order as the experimental work involved in the process of their determination.

The probable error, Q, in r may be computed by the following equation:

$$Q = 0.675 \cdot \frac{1 - r^2}{\sqrt{n}}. \qquad (15)$$

In this particular case $Q = .0098$, which is remarkably low, indicating that the correlation coefficient is very reliable.

By applying a least square method to the values of DS_p and $(\phi_n \cdot \phi_p)$ the equation of the straight line as established by experimental observation may be computed. The usual method employed for this purpose is one which requires that the straight line be so placed with respect to the observed points that the sum of the square of the ordinate deviations ($\Sigma\Delta y^2$) shall be a minimum. Dr. Ludwik Silberstein, of this laboratory, has suggested in this case, where the two quantities concerned are of a similar nature, that it may be preferable to require that the sum of the squares of the shortest distance (that is the perpendicular distance) from the various observed points to the straight line shall be a minimum. This method results in a line which must pass through the centre of gravity of the system (which is not necessarily the case when the $\Sigma\Delta y^2$ is a minimum) and has the additional advantage that the equation of the required straight line may be computed directly from values which have been obtained and used in the computation of the Bravais-Pearson coefficient. The following method for the establishment of the straight line which fulfills the condition that the sum of the squares of the shortest distance, h, shall be a minimum, has been developed by Doctor Silberstein.

If $a = x - \bar{x}$ and $b = y - \bar{y}$, the equation of the straight line (l) satisfying the condition that Σh^2 (where h is the shortest distance from the points x, y to the line l) shall be a minimum is

$$b = m \cdot a \tag{16}$$

where

$$m = \frac{M}{L + \lambda} = \frac{N + \lambda}{M} \tag{17}$$

in which $L = \Sigma a^2$

$M = \Sigma ab$

$N = \Sigma b^2$

and

$$\lambda = -\frac{1}{2}(L + N) + \sqrt{M^2 + \frac{1}{4}(L - N)^2}. \tag{18}$$

It will be observed that the values of L, M, and N are the summations of the last three columns of Table XXXIX, which

illustrates the computation of the Bravais-Pearson correlation coefficient. In terms of x and y the line (16) has the equation

$$y = m \cdot x + (\bar{y} - m\bar{x}) . \qquad (19)$$

The values of $\bar{y}$ and $\bar{x}$ are derived directly from the data in Table XXXIX.

Applying this method to the observed values of DS_p and $(\phi_n \cdot \phi_p)$, we obtain,

$$m = 1.42$$
$$k = .026 = (\bar{y} - m\bar{x}) .$$

Theoretically, these values should be

$$m = 1.43 = BS_0$$
$$k = 0 .$$

The difference between the values obtained by applying the least square method (Σh^2 = a minimum) to the observations and the theoretically correct values are of course attributable to accumulated experimental errors and do not in any way depend upon the statistical observations. These differences will serve as a useful means of estimating the probable errors applying to the statistical relationships about to be considered.

IV. THE CORRELATION BETWEEN THE LIMITING GRADIENT (MINIMUM GRADIENT) OF REPRODUCTION, $e\theta_p$, AND THE DENSITY SCALE OF THE OPTIMAL POSITIVE, DS_p.

From the consideration of the fundamental psychophysical laws of vision it seems logical to expect that a reproduction, in which the minimum gradient has been decreased beyond a certain limiting amount, will not be tolerated as good photographic quality. It is apparent from an inspection of the characteristic curves of the positive materials on which the points representing the maximum and minimum density values of the optimal prints have been located that a considerable portion of this characteristic which is not a straight line has been utilized in almost every case. This of course means that at the extremities of the utilized portion the gradient ($dD/d \log E$) is appreciably less than on the straight-line portion of the characteristic. In beginning an analysis of these results attention was first turned, therefore, to the absolute values of these limiting gradients, $e\theta_p$, as shown in Table XXXVIII. It was at once apparent that these limiting gradient values varied over a considerable range, increasing in any group of similar

papers as the contrast increased and also increasing from group to group as the maximum density (and incidentally the gloss) of the group increased. This tendency is forcibly illustrated by the values of $e\theta_p$ for the first three groups of positive materials. Now an examination of the mean gradient of the optimal negatives, ϕ_n, in the third column of Table XXXVIII, shows that these values tend to vary in a manner approximately the inverse of the variation in $e\theta_p$, while the values of mean gradient for the optimal positives ϕ_p appear to vary in general in the same manner as those of $e\theta_p$. From the standpoint of the visual impression obtained when looking at a positive, the compression of the object brightness scale in the extreme high lights and shadows is determined by the product of $e\theta_p$ by ϕ_n, that is, by the limiting gradient of the reproduction curve, $e\theta_r$. It will be noted that the values of $e\theta_r$ do not vary widely, but there is a marked tendency for an increase in this value from group to group as the maximum density of the materials increases. Now it is not logical to expect that any correlation between a characteristic of the reproduction and a characteristic of the positive material itself shall exist unless that characteristic of the positive material be also a characteristic of the portion of the curve which is actually used in making the reproduction. It does not seem justifiable therefore to attempt to correlate $e\theta_r$ with the maximum density of the positive material, but it does seem perfectly justifiable to expect some relationship to exist between this factor, $e\theta_r$, and the density scale of the optimal positive which, so far as the particular reproduction is concerned, is the maximum density value. $e\theta_r$ for each of the materials used was therefore plotted as a function of the density scale of the optimal positive, DS_p. The result is shown in Fig. 54. It will be noted that there is appreciable scattering of the points from a straight-line relationship although there is a very definite correlation between the two values. Applying the method previously described in detail the correlation coefficient was computed, the results obtained being as follows:

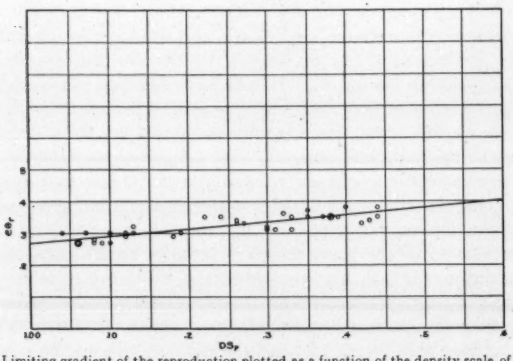
$$\text{Correlation coefficient } (r) = .87.$$
$$\text{Probable error } (Q) = .04.$$

It was shown in the previous section that, due to the presence of experimental uncertainties, a correlation coefficient greater than .95 cannot be expected. The value obtained above, which measures

the degree of correlation between $e\theta_r$ and DS_p, approaches quite closely to the maximum value which can be expected. It is apparent, therefore, that there is a very definite correlation between these two factors.

An inspection of the plotted points (Fig. 54) indicates that the relation between $e\theta_r$ and DS_p may be represented very well by a a straight line. Applying the method of least squares to the observations, the constants in the equation of the straight line satisfying the condition that the sum of the squares of the shortest

FIG. 54.



Limiting gradient of the reproduction plotted as a function of the density scale of the optimal positive.
$e\theta_r = .23\ DS_p + .036.$

distance from each observed point to the line shall be a minimum are:

$$m = .23$$
$$k = .036.$$

Therefore $$e\theta_r = .23 \cdot DS_p + .036. \qquad (20)$$

One of the objects in seeking a definite correlation between the statistically determined characteristics of the optimal positives and negatives and the sensitometrically measured characteristics of the positive materials is the establishment of some definite rule for choosing a paper (positive material) on which to print a given negative or as a guide in making a negative suitable for printing on some specified positive material. The evidence in the present

case indicates there is a definite correlation between the limiting gradient of the reproduction and the brightness scale of the reproduction (BS_r), which is of course equivalent to the density scale of the positive. This result seems entirely logical when considered from the standpoint of our knowledge of the psychophysical laws of vision and visual requirements. The limiting gradient of the reproduction designates the slope at the extremity of the reproduction curve and is of the form

$$\frac{dD_p}{d \log B_0}.$$

This when applied to finite increments assumes the form

$$\frac{\Delta D_p}{\Delta \log B_0}.$$

For D_p in these ratios we may substitute $\log B_r$. It is evident, therefore, that the gradient of the reproduction curve at any point is merely a ratio which indicates the relation between a brightness difference in the object and the corresponding brightness difference in the reproduction. If $e\theta_r$ is less than unity it indicates that a given brightness difference in the object is rendered by a smaller brightness difference in the reproduction, and since it is usually necessary to utilize as much as possible of the total available density scale of the positive material this limiting gradient usually represents the greatest compression of brightness differences in the shadows and high lights of the object that can be tolerated as acceptable photographic quality. The value of $e\theta_r$ is a definite function of the density scale of the positive as shown by equation (20). For instance, if

$$BS_r = 1.00, \quad e\theta_r = .27.$$

While if

$$BS_r = 1.40, \quad e\theta_r = .36.$$

In the first case an object brightness difference, $\Delta \log B_0$, in the extreme high-light and shadow regions, of .1 is represented by a brightness difference in the reproduction of .027, while in the second case a brightness difference in the reproduction of .036 is demanded for satisfactory reproduction of the same brightness difference (.1) in the object. It seems reasonable to expect such a result, for when the difference between the maximum and minimum brightness in the field is great, the brightness difference between two parts of that field relatively close together on the

brightness scale must be greater, in order to have the same subjective magnitude relative to the total brightness scale, than in case of a visual field in which the total brightness scale is less.

To determine the negative gradient, ϕ_n, which for an object brightness scale of 1.44 will yield a reproduction fulfilling the requirements of the statistical equation (20) on any specified positive material.—The magnitude of the correlation coefficient indicates that the relation $e\theta_r = m \cdot DS_r + k$ holds with considerable precision for the particular test-object used in this work. It will be recalled that the value of k in the equation of the straight line representing the relationship between the density scale of the positive, DS_p, and the product $\phi_n \cdot \phi_p$ obtained in a similar manner was .036 and further that this theoretically should be 0. This indicates that the value of k is uncertain by the amount of this difference. In the case of the relation between $e\theta_r$ and DS_r it is therefore doubtful whether the value of $k = .036$ is significant. It is probable that a line drawn through the origin, $k = 0$, will represent the relationship between $e\theta_r$ and DS_r to within limits of the experimental errors involved. A straight line drawn through the origin and through the centre of gravity of the observed points has a slope of .25. The difference between this and the value actually determined (.23) is also within the limits of experimental error. For all practical purposes it seems reasonable to assume that the equation

$$e\theta_r = .25\, DS_p \qquad (21)$$

represents the facts as satisfactorily as the relation actually determined by the least square method.

Now it is possible to compute for any positive material of which the D-log E characteristic is known, the gradient of the negative that will give the best possible rendering of the object used in this work or, it seems reasonable to assume, any object having the same brightness scale, that is 1.44.

The method of obtaining the desired value of ϕ_n is illustrated by the curves in Fig. 55. The necessary data for plotting the three curves, A, B, and C in this figure, are obtained directly from the sensitometric data relating to the positive material in question. The D-log E characteristic is plotted in the usual manner as illustrated by curve A in Fig. 31, Section D. From this the curve B (in the same figure), which is the first derivative of

A, is plotted in the manner already explained. A horizontal line $zs'r'$ drawn through any value of gradient cuts the first derivative B at the points s' and r'. Perpendiculars through these points establish the points r and s on the D-log E characteristic. The difference between the ordinate values of these points (s, r) gives the density scale of that portion of the positive characteristic for which the equalized limiting gradient is the ordinate of the line $zs'r'$. Likewise the difference between the

FIG. 55.

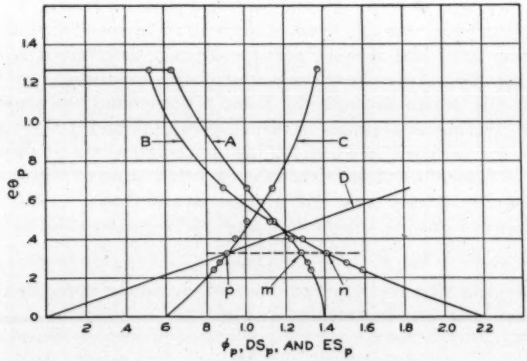


Illustration of a graphic method for finding the negative gradient which will satisfy the statistical relationship of equation (21) when using a positive material of which the D-log E characteristic is known and object brightness scale is equal to 1.44.

abscissa values of these points (s, r) gives the exposure scale corresponding to the same equalized limiting gradient. The ratio of density scale to exposure scale gives the mean slope ϕ_p of a positive having the same limiting gradient, $e\theta_p$. In this way values of DS_p, ES_p, and ϕ_p can be determined for any assumed values of $e\theta_p$. By determining these values for all values of $e\theta_p$ between 0 and γ_y, the data from which the curves shown in Fig. 55 are plotted may be obtained. Curve A shows DS_p as a function of $e\theta_p$, curve B shows the relation between ES_p and $e\theta_p$, while curve C shows the mean gradient of the positive, ϕ_p, as a function of $e\theta_p$.

Now by substituting for $e\theta_r$ in equation (21), its equivalent $e\theta_y \cdot \phi_n$, we obtain

$$e\theta_y \cdot \phi_n = .25\, DS_r$$
$$\phi_n = DS_n/BS_0 = ES_p/BS_0$$
$$DS_r = DS_p$$
$$\therefore \quad e\theta_y = .25\, BS_0\,(DS_p/ES_p)$$
$$= .25\, BS_0\,(\phi_p)\,. \qquad (22)$$

It is evident that this is also an equation of a straight line passing through the origin and having a slope equal to $.25\ BS_0$, which in this case is equal to .36. Curve *D* in Fig. 55 represents this function, having a slope of .36 and passing through the origin. The point *p* at which the line *D* cuts the curve *C* satisfies the statistical equation (21) and also the sensitometrically determined relation between $e\theta_p$ and ϕ_p for the assumed positive material. A horizontal line drawn through the point *p* determines the points *m* and *n*, the abscissa values of which give ES_p and DS_p, respectively, for the positive which fulfills the requirements of equation (21). Remembering now that the exposure scale of the print is equal to the density scale of the negative and that

$$\phi_n = \frac{DS_n}{BS_0},$$

it is evident that the required value of ϕ_n can be computed. In the particular case represented in Fig. 55,

$$ES_p = 1.41$$
$$\therefore \quad \phi_n = \frac{1.41}{1.44} = .98\,.$$

Therefore a negative developed to a gamma of .98 will give a reproduction on the positive material fulfilling the requirements of the statistical equation (21).

The application of the statistical relationship shown by equation (20) to the reproduction of objects varying in brightness scale.—If it be assumed that this statistical relationship is valid for other values of BS_0, the characteristics of the negative to satisfy any positive material of known sensitometric characteristics can be deduced for objects of any specified brightness scale or conversely it is possible to determine the brightness scale of an object which for any specified negative gradient (ϕ_n) will be rendered in such a way as to satisfy the requirements of equation (20).

It is of interest to apply this solution to a few typical positive materials selected to cover the range of contrast available in these materials. For this purpose the positive materials designated as 1, 23, 4, 8, 9, 10, 11, and 21 have been selected, this being the same group that has been used previously for illustrative purposes

FIG. 56.

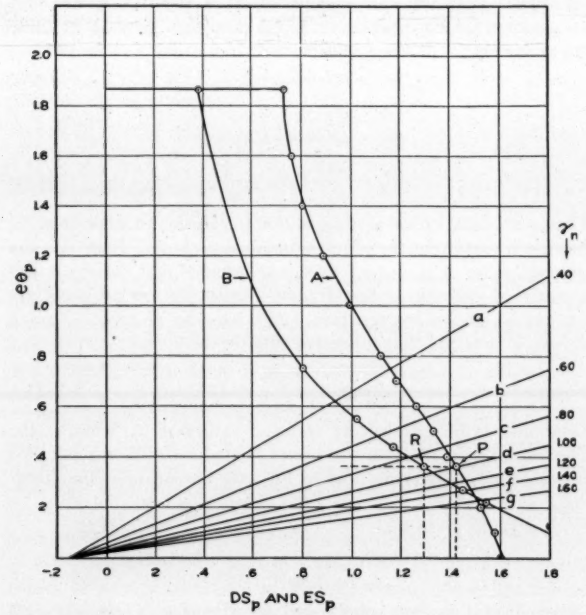


Illustration of a graphic method for finding values of negative gamma and object brightness scale which will meet the requirements of statistical equation (20), any specified positive material being used.

(see Figs. 32 to 39 and 41 to 48, Section D). In order to compute the desired value it is necessary to evaluate the expression $DS_y = f(e\theta_y)$. Since no satisfactory analytical expression is available for expressing density as a function of the logarithm of the exposure, recourse must again be had to a graphic solution. The method is illustrated in Fig. 56, which applies to positive

material No. 9. Curve A shows the relation between $e\theta_y$ and DS_y, while curve B shows ES_y as a function of $e\theta_y$ for the material under consideration. The data for these curves are obtained directly from the sensitometric measurements as explained in reference to the construction of Fig. 55.

Now let it be required to determine the brightness scale of the object which for some specified value of negative gamma may be rendered on the positive material in such a way as to fulfill the requirements of the equation

$$e\theta_r = m \cdot DS_p + k \qquad (20)$$

$$e\theta_r = e\theta_p \cdot \phi_n$$

$$\therefore \quad e\theta_p \cdot \phi_n = m \cdot DS_p + k$$

$$e\theta_p = \frac{m}{\phi_p} \cdot DS_p + \frac{k}{\phi_n} \cdot \qquad (23)$$

This is an equation of a straight line in which the slope is m/ϕ_n and the intercept on the $e\theta_p$ axis is equal to k/ϕ_n. Now for any assumed value of ϕ_n (or γ_n, since it is assumed that the object can be rendered entirely on the straight-line portion of the material) the curve defined by equation (23) may be plotted. Placing $e\theta_p = 0$, it is found that the curve cuts the x-axis at

$$-\frac{k}{m} = -\frac{.036}{.23} = .156.$$

This value is independent of assumed values of ϕ_n, hence is the intersection point of all of the curves plotted for different assumed values of ϕ_n. Referring now to Fig. 56, the straight line designated as d satisfies equation (20) when $\phi_n = 1.00$, the slope of this line being given by,

$$\frac{m}{\phi_n} = \frac{.23}{1.00} = .23 .$$

Curve d is therefore established by drawing a straight line through the point O, at $DS_p = -.156$, with the slope of .23. Now the values of $e\theta_y$ and DS_p for the point, P, at which the curve d cuts curve A, satisfies the equation

$$DS_y = f(e\theta_y)$$

for the positive material under consideration (No. 9) and also the statistical equation

$$e\theta_r = .23\, DS_p + .036 . \qquad (20)$$

A perpendicular dropped from the point P onto the x-axis determines the values of DS_p, which is the density scale of the positive made on material No. 9 from a negative having a gamma of unity and satisfying the statistical equation above. A horizontal line drawn through the point P intersects curve B at point R. The abscissa value of the point R gives the value of the exposure scale of the positive satisfying the requirements. The exposure scale of the positive must be identical with the density scale of the negative and since the negative being used has a gamma of unity, the brightness scale of the object may be computed by the equation,

$$BS_0 = \frac{DS_n}{\gamma_n}.$$

In this particular case,

$$BS_0 = \frac{1.29}{1.00} = 1.29.$$

The requirements of the statistical equation (20) are therefore fulfilled when the object brightness scale is 1.29 and the negative is developed to a gamma of unity. Insofar as satisfactory rendering is defined by this statistical relation, a positive made on this material from such a negative, BS_0 being equal to 1.29, should give acceptable photographic quality.

It is apparent, if the negative be developed to a gamma other than unity, that an object of different brightness scale will be required in order to fulfill the conditions of equation (20). By assuming other values of gamma a series of curves similar to d can be drawn as shown in the figure. All of these must pass through the point O and their slopes are determined by the value of m/ϕ_n. The curves a, b, c, etc., are drawn for various assumed values of gamma as shown at the right-hand side of the figure. By proceeding in a manner similar to that already described, the density scale of the positive, the exposure scale of the positive, and the brightness scale of the object required to fulfill the conditions of equation (20) may be determined.

In Table XL are shown the results, for a group of eight positive materials increasing progressively in contrast, obtained by using the assumed values of gamma corresponding to the curves a, b, c, etc., in Fig. 56. The values of $e\theta_p$ are read directly from the graphic solution by reading off the ordinate value of the points

TABLE XL.

Characteristics of photographic reproductions obtained by extending statistical equations (20) and (24) to the cases where the brightness scale of the object is variable.

Material No. 1: D_y max. = 1.15; γ_y = .83; Ω_y = .72.

γ_M.	$e\theta_p$.	DS_p.	ES_p.	BS_o.	ϕ_r.	γ_r.	$e\theta_r$ (20).	$e\theta_r$ (24).
.4	.54	.78	1.01	2.53	.31	.33	.22	.20
.6	.39	.92	1.30	2.17	.42	.50	.23	.22
.8	.33	.99	1.44	1.80	.55	.66	.26	.24
1.0	.27	1.03	1.59	1.59	.65	.83	.27	.26
1.2	.23	1.05	1.72	1.43	.74	1.00	**.28**	**.28**
1.4	.20	1.07	1.82	1.30	.82	1.16	.28	.30
1.6	.18	1.09	1.93	1.20	.91	1.33	.29	.32

Material No. 23: D_y max. = 1.21; γ_y = .94; Ω_y = .95.

.4	.61	.92	1.04	2.60	.42	.38	.24	.21
.6	.45	1.03	1.23	2.06	.50	.56	.27	.23
.8	.35	1.09	1.38	1.73	.63	.75	.28	.25
1.0	.29	1.12	1.48	1.48	.76	.94	.29	.28
1.2	.25	1.14	1.56	1.30	.88	1.13	**.30**	**.30**
1.4	.21	1.15	1.63	1.16	.99	1.31	.30	.32
1.6	.19	1.16	1.68	1.05	1.10	1.50	.30	.35

Material No. 4: D_y max. = 1.40; γ_y = 1.13; Ω_y = 1.28.

.4	.69	1.05	1.02	2.55	.41	.45	.28	.22
.6	.50	1.16	1.22	2.04	.57	.68	.30	.24
.8	.40	1.23	1.37	1.71	.72	.90	.32	.27
1.0	.33	1.27	1.49	1.49	.85	1.13	.33	.30
1.2	.28	1.30	1.58	1.32	.99	1.36	**.34**	**.33**
1.4	.24	1.32	1.68	1.20	1.10	1.58	.34	.36
1.6	.21	1.33	1.76	1.10	1.21	1.81	.34	.38

Material No. 8: D_y max. = 1.52; γ_y = 1.52; Ω_y = 1.58.

.4	.74	1.14	.99	2.57	.43	.43	.30	.21
.6	.54	1.26	1.23	2.05	.61	.64	.32	.24
.8	.43	1.33	1.41	1.76	.76	.86	.34	.27
1.0	.35	1.38	1.56	1.56	.88	1.07	.35	.29
1.2	.30	1.41	1.65	1.37	1.03	1.28	.36	.32
1.4	.26	1.44	1.74	1.24	1.16	1.50	.36	.35
1.6	.23	1.45	1.81	1.13	1.29	1.71	**.37**	**.37**

TABLE XL (*Continued*).

Material No. 9: D_y max. = 1.60; γ_y = 1.33; Ω_y = 2.03.

γ_n.	$e\theta_p$.	DS_p.	ES_p.	BS_o.	ϕ_r.	γ_r.	$e\theta_r$ (20).	$e\theta_r$ (24).
.4	.75	1.16	.80	2.20	.52	.53	.30	.23
.6	.55	1.29	1.02	1.70	.76	.80	.33	.26
.8	.44	1.38	1.17	1.46	.95	1.06	.35	.29
1.0	.36	1.42	1.29	1.29	1.10	1.33	.36	.33
1.2	.31	1.46	1.38	1.15	1.27	1.60	**.37**	**.36**
1.4	.27	1.48	1.45	1.04	1.42	1.86	.38	.39
1.6	.24	1.51	1.54	.96	1.57	2.13	.38	.42

Material No. 10: D_y max. = 1.67; γ_y = 1.73; Ω_y = 2.79.

.4	.83	1.30	.75	1.87	.69	.69	.33	.25
.6	.60	1.42	.93	1.55	.92	1.04	.36	.29
.8	.47	1.49	1.08	1.35	1.10	1.38	.38	.33
1.0	.39	1.52	1.19	1.19	1.29	1.73	**.39**	**.37**
1.2	.33	1.55	1.29	1.07	1.45	2.08	.40	.42
1.4	.29	1.57	1.37	.98	1.62	2.42	.41	.46
1.6	.25	1.59	1.44	.90	1.77	2.77	.41	.51

Material No. 11: D_y max. = 1.70; γ_y = 2.08; Ω_y = 3.46.

.4	.85	1.34	.64	1.60	.84	.84	.34	.26
.6	.62	1.47	.81	1.35	1.09	1.25	.37	.31
.8	.49	1.53	.93	1.16	1.32	1.66	**.39**	**.37**
1.0	.40	1.58	1.04	1.04	1.52	2.08	.40	.42
1.2	.34	1.60	1.12	.93	1.72	2.50	.41	.47
1.4	.29	1.62	1.19	.85	1.91	2.91	.41	.52
1.6	.26	1.64	1.24	.78	2.10	3.32	.42	.56

Material No. 21: D_y max. = 1.52; γ_y = 2.48; Ω_y = 3.69.

.4	.84	1.31	.55	1.37	.91	.99	.34	.27
.6	.59	1.39	.68	1.13	1.23	1.24	.35	.31
.8	.46	1.44	.76	.95	1.52	1.98	**.37**	**.41**
1.0	.37	1.46	.82	.82	1.78	2.48	.37	.47
1.2	.31	1.47	.87	.72	2.04	2.98	.37	.53
1.4	.27	1.48	.90	.64	2.31	3.48	.38	.59
1.6	.24	1.49	.93	.58	2.57	3.97	.38	.65

where the straight lines cut the curve A. The abscissa values of these same points give the corresponding density scales, DS_p, and horizontal lines through the intersection points determine the points on curve D whose abscissa values are the required exposure scales, ES_p. Since $BS_0 = \frac{DS_n}{\phi_n}$, the value of BS_0 can be computed as shown in the fifth column of the table. This is the brightness scale of an object which for the negative gamma assumed fulfills the requirements of the statistical equation (20). Values in the column ϕ_r are derived by dividing DS_p by BS_0. The maximum slope of the reproduction, γ_r, is obtained by multiplying γ_y for the positive material by γ_n, and the minimum slope $e\theta_r$ is the product of $e\theta_p$ by γ_n. In order to indicate that these values of minimum slope are derived from the statistical equation (20), this number is placed in parentheses after the symbol at the top of the column. The numbers in any horizontal line therefore specify the characteristics of the positive, the object, and the reproduction which satisfy equation (20). It will be noted that as the negative gamma increases, the density scale of the positive increases and approaches the maximum density (D_y max.) of the positive material, while the brightness scale of the object decreases progressively with increasing negative gamma. It should be remembered that these values are based on the assumption that the constants in equation (20), determined by observations based on a single object, are valid for objects of different brightness scale. It will be noted also that the greater the value of object brightness scale, the greater is the value of $e\theta_p$ and hence the greater must be the sacrifice of the available density scale of the material in order to conform to the requirements of the statistical equation (20). As will be shown in the next section, there are other limitations for satisfactory photographic rendering than are contained in equation (20), and hence the conclusions drawn from the data in Table XL must be modified to a certain extent by the additional requirements of other relationships.

V. CORRELATION BETWEEN THE LIMITING GRADIENT (THAT IS MINIMUM GRADIENT) OF THE REPRODUCTION AND THE MAXIMUM GRADIENT OF THE REPRODUCTION, γ_r.

There is another point of view from which satisfactory photographic quality may be regarded. The necessity of utilizing portions of the positive characteristic which have gradients appre-

ciably less than unity must be recognized. It might be argued that by making the gamma of the negative sufficiently high, the gradient at the limit of the reproduction curve could be raised materially and thus avoid an intolerable diminution in the reproduction of a brightness difference in the shadow and brightness regions. Of course this could only be applied provided the brightness scale of the object under consideration is sufficiently small so that the use of a high negative gamma would not give a negative density scale greater than the available exposure scale of the positive material. But even in such a case the use of high negative gamma would result in a high value of maximum gradient in the reproduction curve. It will be remembered that the gradient at any point on the reproduction curve is given by the product of the gradients at corresponding points on the characteristic curve of the positive and negative. The maximum gradient of the reproduction is therefore obtained by multiplying the gamma, γ_y, of the positive material by the gradient of the negative (continuing, of course, the assumption that the entire brightness scale is rendered on the straight-line portion of the material). The minimum gradient of the reproduction is obtained by multiplying the limiting gradient of the positive by the gradient of the negative. Thus

$$G_r \text{ max.} = \gamma_y \cdot \phi_n$$
$$G_r \text{ min.} = e\theta_p \cdot \phi_n = e\theta_r .$$

In Table XLI the values of these maximum and minimum reproduction gradients are given. An examination of these shows at once that there is an indication of rather good correlation. The Bravais-Pearson coefficient was computed from these observations according to the method previously described with the following results:

Correlation coefficient, $r = .73$.
Probable error, $Q = .035$.

This correlation, while not as high as that found for the relation between the equalized limiting gradient of the reproduction and the brightness scale of the reproduction, is sufficiently high to indicate a very definite relationship between the maximum and minimum gradient of the reproduction. That the eye demands some such relationship as a criterion of satisfactory photographic rendering is entirely logical and quite in accord with our general

TABLE XLI.

Corresponding values of maximum and minimum reproduction gradients, as shown graphically in Fig. 57.

No.	G_r min.	G_r max.	No.	G_r min.	G_r max.
1	.27	1.06	20	.27	1.13
2	.30	1.03			
3	.27	1.18	21	.31	1.28
4	.31	1.30	22	.35	1.23
5	.34	1.36	23	.29	1.02
6	.33	1.62	24	.29	1.00
7	.30	1.41	25	.29	1.14
8	.36	1.61	26	.37	1.06
9	.38	1.76			
10	.38	1.56	27	.31	1.14
11	.35	1.44	28	.36	1.31
			29	.32	1.37
12	.38	1.52			
13	.35	1.42	30	.32	1.06
			31	.35	1.19
14	.30	1.29	32	.33	1.22
15	.29	1.07			
			33	.27	1.04
16	.37	1.72	34	.34	1.27
17	.35	1.36	35	.29	1.37
			36	.35	1.31
18	.33	1.62	37	.28	1.07
19	.35	1.24			

knowledge of visual requirements. It is evident from the form of the equation that a smaller value of gradient $\left(\frac{d \log B_r}{d \log B_0}\right)$ at the upper and lower limits of the reproduction curves, that is the high-light and shadow regions, is tolerated as satisfactory photographic rendering in the case of a reproduction in which the maximum gradient is relatively low than when this maximum gradient is high. It follows, therefore, that the brightness difference in the reproduction which is considered as a satisfactory minimum for representing a unit brightness difference in the object cannot be specified by any absolute value but is dependent upon the maximum brightness difference in the reproduction corresponding to the unit brightness difference in the object.

Applying again the method of least squares in which Σh^2 is made a minimum, the constants of the straight line which fits most perfectly the observed values are found to be

$$m = .124$$
$$k = .16 .$$

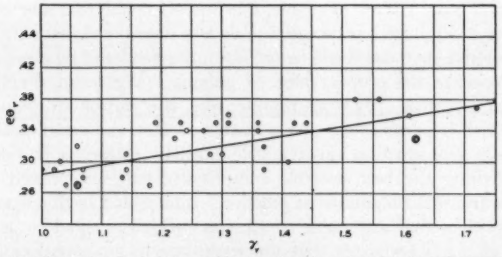
The equation of this line therefore becomes,

$$e\theta_r = .124\,\gamma_r + .16 . \qquad (24)$$

In Fig. 57 the straight line is drawn so as to fit equation (24) and the small circles show the points obtained by plotting the observed values given in Table XLI.

If now the requirements of equation (24), which seems to be determined with considerable certainty by the statistical observations, be added to those established by equation (20), which

FIG. 57.



Equalized limiting gradient of the reproduction plotted as a function of maximum gradient of the reproduction.
$e\theta_r = .124\,\gamma_r + .16.$

also seems to have great validity both from the standpoint of theoretical visual requirements and experimental statistical observations, a greater limitation is imposed upon the adjustment of negative gamma and object brightness scale to the sensitometric characteristics of the positive material. It is evident that if the requirements of equation (24) are to be met, the rendition of objects varying in brightness scale on a given material as indicated by the data in Table XL cannot be tolerated in all cases. By the use of equation (24) the values of $e\theta_r$ required for any assumed value of γ_r may be computed. In Table XL the column designated as γ_r contains the values of this factor which fulfills the requirements of the statistical equation (20) for the γ_n values in the first column. Now by inserting these values of γ_r in equation (24), corresponding values of $e\theta_r$, as demanded by this second condition, were computed and are shown in the last column of Table XL

designated as $e\theta_r$ (24). Now it will be observed that for low values of γ_n there is a tendency for $e\theta_r$ (24) to be less than $e\theta_r$ computed for equation (20). For intermediate values of γ_n the two values of $e\theta_r$ become equal, and when γ_n is high the ratio of $e\theta_r$ (20) to $e\theta_r$ (24) is reversed.

From a general consideration of the problem of tone reproduction it is evident that there is no particular reason why the gradient at the limits of the reproduction curve should be less than the maximum gradient, provided positive materials of such a nature as to permit such reproduction were available. For instance, assuming that a positive material having a perfect straight-line characteristic throughout the entire density scale (0 to D max.) were available, there is no doubt that any object having a brightness scale equal to or less than the density scale of the positive material could be satisfactorily rendered by a negative developed to the proper value of gamma. We must, therefore, regard equation (24) as representing the minimum value of $e\theta_r$ which is permissible. As a matter of fact, the entire problem of tone reproduction on available positive materials is one of establishing the best possible compromise between perfect rendition and that obtainable in practice. Interpreted in this manner, it is evident that any value of $e\theta_r$ (24) which is equal to or less than $e\theta_r$ (20) indicates that the requirements of equation (24) are satisfactorily fulfilled by using negative gamma and BS_n values which satisfy equation (20). In Table XL values of $e\theta_r$ shown in bold face type indicate the maximum γ_n value which meets the requirements of both equations. It will be noted that as the contrast of the material increases this maximum gamma value decreases. Such a result is in accord with practical experience.

It is common practice in some kinds of photographic work to develop all negatives in a standardized developer for a fixed time, thus giving the same value of γ_n regardless of the brightness scale of the object. In making prints from such negatives, the positive material is chosen from the standpoint of contrast to give a satisfactory result from these negatives which obviously must vary appreciably in density scale. The extent to which development is carried depends somewhat on the class of work being done. It is probable that a gamma of .8 is fairly representative for portrait negatives, while a gamma of 1.0 represents approximately

the general average used in amateur finishing work. It is of interest to analyze the data obtained in the last two parts of this paper from the standpoint of objective brightness scale, BS_0, which is satisfactorily rendered on various positive materials by use of negative gammas mentioned above, which may be assumed to represent approximately two important classes of photographic finishing.

For each of the positive materials used complete computation

TABLE XLII.

Values of objective brightness scale, BS_0, which for $\gamma = .8$ and $\gamma = 1.0$ meet the requirements of statistical equations (20) and (24) when using the various positive materials.

No.	$\gamma = .8$.	$\gamma = 1.0$.	No.	$\gamma = .8$.	$\gamma = 1.0$.
1	1.80	1.59	20	1.53	1.31
2	1.75	1.53			
3	1.19	1.03	21	.95	....
4	1.71	1.49	22	1.61	1.38
5	1.61	1.41	23	1.73	1.48
6	1.32	1.16	24	1.72	1.49
7	1.20	1.07	25	1.63	1.41
8	2.05	1.76	26	1.70	1.46
9	1.70	1.46			
10	1.55	1.35	27	1.45	1.25
11	1.35	1.16	28	1.49	1.31
			29	1.20	1.06
12	1.70	1.44			
13	1.38	1.20	30	1.66	1.48
			31	1.71	1.47
14	1.59	1.36	32	1.61	1.41
15	1.34	1.17			
			33	1.81	1.55
16	1.58	1.40	34	1.68	1.45
17	1.28	1.09	35	1.71	1.53
			36	1.53	1.32
18	1.25	1.12	37	1.60	1.40
19	1.12	.97			

of data similar to that shown for a few of the materials in Table XL was made. From these the BS_0 values corresponding to negative gammas of .8 and 1.0 were read. These values are shown in Table XLII. It was found that for all materials, with the exception of No. 21, a negative gamma of .8 is permitted by the requirements of equations (20) and (24), and in the case of that material $e\theta_r$ (24) is but slightly greater than $e\theta_r$ (20). It is assumed that a gamma of .8 can be used for this material

without serious departure from the two statistically determined relationships. Likewise it was found that a negative gamma of 1.0 is permissible with the exception of a few cases of the more contrasty materials. The difference between $e\theta_r$ (20) and $e\theta_r$ (24) is sufficiently great in only one case to prohibit the use of such a negative gamma. This is in case of material No. 21, for which no value corresponding to $\gamma_n = 1.0$ is shown in the table. It should be kept in mind that the validity of these values shown in this table and also in Table XL is dependent upon the assumption that the constants in equations (20) and (24), which were determined experimentally by use of a test-object having a brightness scale of 1.44, apply also to the rendition of objects having different brightness scales. The author considers it probable that this assumption is permissible, although further experimental work must be done before final conclusions can be drawn.

The values in Table XLII show that the positive materials of relatively low contrast and high exposure scale permit the rendition of objects for which the brightness scale is relatively great; while the more contrasty papers, having relatively short exposure scales, are adapted to the rendition of objects in which the brightness scale is low. However, the brightness scale values as shown in Table XLII do not correlate directly either with total contrast of the positive material or with the total brightness scale of the positive material. Here again it is apparent that the proportion of total available density scale which can be used in any particular case is dependent upon *gradient conditions*, and the proportion of total density scale which must be sacrificed varies for different materials and depends upon *values other than the sensitometric characteristics of the positive material itself*.

VI. CORRELATION BETWEEN THE EQUALIZED LIMITING GRADIENT OF THE OPTIMAL POSITIVE, $e\theta_p$, AND THE MAXIMUM GRADIENT OF THE POSITIVE, γ_p.

A comparison of the values of $e\theta_p$ in Table XXXVIII with those of γ_y in Table XXXVII suggests the existence of some relationship since the two values tend to show corresponding variations in magnitude. It seems desirable, therefore, to study this correlation in detail. In Fig. 58 the observations are plotted. It is evident that a straight line fits quite satisfactorily the

observed values. The Bravais-Pearson correlation coefficient was computed and found to be very high,

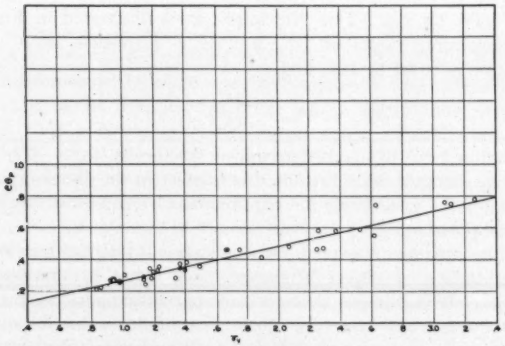
$$r = .97$$

with a probable error

$$Q = .007 .$$

This correlation is surprisingly high, in fact slightly higher than the correlation found previously (see Part III of this section) between two factors related theoretically so as to require correlation coefficient of 1.00; but which actually, due to experimental

FIG. 58.



Equalized limiting gradient of the optimal positives plotted as a function of maximum gradient, $e\theta_p = .22\,\gamma_p + .041$.

errors, gave only .95. The evidence indicates, therefore, that the correlation between the two factors being considered at present is as good as can be expected from the experimental work.

Applying the method of least squares, making Σh^2 minimum, the following values for the constants in the equation of the straight line were found:

$$m = .22$$

$$k = .041 .$$

The equation of the straight line (Fig. 58) most closely fitting the observations becomes,

$$e\theta_p = 22\,\gamma_p + .041 , \qquad (25)$$

γ_p being equal to γ_v.

For the test-object used in this work, which has a brightness scale of 1.44, and for the group of positive materials studied, equation (25) holds with a very high degree of precision and it seems entirely justifiable to assume that for any object having a similar brightness scale the equation can safely be applied to the computation of the conditions giving satisfactory photographic quality. In order to determine for any photographic material, for which the D-log E characteristic is known, the relationship between negative gamma and object brightness scale which will meet the requirements of equation (25), it is only necessary to insert in the equation the value of positive gamma and solve for $e\theta_p$. This determines, as is illustrated in Fig. 56, corresponding values of DS_p and ES_p. Exposure scale of the positive is equal to DS_n and,

$$\gamma_n = \frac{DS_n}{BS_0}.$$

Equation (25) has been determined specifically for a BS_0 value of 1.44, hence if we substitute this number in the above equation, the value of γ_n meeting the requirements of equation (25) may be computed.

The attempt to extend the application of equation (25) to other values of object brightness scale leads to unreasonable results. It is obvious that for any particular positive material γ_p is constant and hence the substitution of this value in equation (25) gives a value of $e\theta_p$ which is independent of object brightness scale. This means that for any positive material a fixed proportion of the total density scale is available for use regardless of the object brightness scale. The statistical equations (20) and (24) indicate that the useful part of the total density scale of any material is determined by considerations of *limiting gradient* and hence must be dependent upon the slope of the negative and indirectly upon the brightness scale of the object. It is evident, therefore, that equation (25) cannot be extended to objects having an object brightness scale other than 1.44 without violating the requirements of equations (20) and (24), both of which seem to be established with a fair degree of certainty. Moreover, we know from practical observations that the available portion of the density scale of any positive material depends to some extent at least on the brightness scale of the object being

reproduced. There is little doubt, therefore, that the values of constants in equation (25) must be in some way dependent upon the brightness scale of the object. It seems entirely unjustifiable, therefore, to assume that m and k in equation (25) should retain the values found in this work when the brightness scale of the object is other than 1.44.

Attention should be called to the similarity in the values of the constants m and k in the two equations (20) and (25). These values are identical to well within the limits of experimental error. No explanation for this is at present offered, although it seems highly improbable that this can be mere coincidence, or that this equality of constants, if it be not merely coincidence, can exist only in the case of objects having the brightness scale equivalent to that of the test-object used in this work. A further study of this problem must be made using test-objects in which the brightness scale varies over a wide range before any definite decision can be made as to the exact significance of the similarity between equations (20) and (25). The correlation coefficient in both cases is so high that it seems extremely unreasonable to conclude that the relationships do not have a real significance. At present a more extensive study of the subject is in progress in which the brightness scale of the object is being varied over wide limits. It is quite possible that the values of the constants in all three of these statistical equations are functions of object brightness scale, and if such a relationship can be definitely established, it will then be possible to specify with considerable certainty the characteristics of a positive material which will give the best possible photographic rendition of an object having a known brightness scale and also the gamma to which the negative must be developed in order to obtain optimal photographic quality.

VII. VARIATION IN GRADIENT OF OPTIMAL NEGATIVES, ϕ_n, FOR GROUPS OF POSITIVE MATERIAL EQUAL WITH RESPECT TO: (a) EXPOSURE SCALE, ES_y; (b) MEAN GRADIENT ϕ_y; AND (c) DENSITY SCALE, DS_y.

The correlations thus far discussed all involve a limiting gradient characteristic of the optimal positive, or reproduction curves. They are somewhat complex and do not provide a very simple or direct method of connecting the characteristics of the negative and positive material combination giving optimal photographic quality. The reader may feel inclined to criticize the treatment of the data on account of its complicated nature. The desirability of finding

a simpler method, preferably involving only the use of values derivable directly from the sensitometric characteristics (Table XXXVII) of the positive material, is keenly appreciated by the author; but a most thorough and painstaking analysis of the data has failed thus far to reveal any correlation of even approximate nature which can be used for the desired purpose.

When this work was started it was expected that some definite relation between the negative gradient (ϕ_n or γ_n) giving the best possible print on a given positive material and the exposure scale of that material would be found. Since the brightness scale of the object is constant for all test negatives used, the density scale of the negative is directly proportional to γ_n, hence the utilization of the total exposure scale of positive materials equal in exposure scale would require optimal negatives of equal gradient. The absence of even an approximate approach to this condition is forcibly illustrated by the data in Table XLIII. By classification of the sensitometric data five groups have been obtained, in each of which ES_y is practically constant. In the first group $ES_y = 1.35 \pm .02$. The values of optimal negative gradient, ϕ_n (equivalent in this case to γ_n), vary from .46 to .72. Within each of the other four groups, for each of which ES_y is constant, the variation in ϕ_n is large. These variations in ϕ_n are many times as great as the probable error involved. For instance, considering the first group in Table XLIII, a print from the negative for which $\phi_n = .54$ made on material No. 29 would not even remotely approach satisfactory photographic quality. It is evident, therefore, that papers of identical exposure scale require, for the most satisfactory rendition of the same object, negatives differing widely in gradient. An inspection of the other factors shown in Table XLIII does not reveal any systematic variation in their magnitude which permits the establishment of a satisfactory correlation (even within a group for which ES_y is constant) between ϕ_n and one or any combination of two or more of the sensitometric characteristics of the positive material.

Failing to find a satisfactory correlation between γ_n and ES_y, attention was turned to the mean gradient of the positive material, ϕ_y. It was noted in a study of Table XLIII that some of the large variations in γ_n for materials of equal exposure scale seem to be accompanied by corresponding variations in density scale. This suggests that possibly the mean gradient of the positive

TABLE XLIII.

Classification of positive materials on the basis of equality in exposure scale, ES_y, illustrating the lack of correlation between the gradient of the optimal negative and the exposure scale values.

No.	DS_y.	ES_y.	ϕ_y.	γ_y.	ϕ_n.
15	1.17	1.32	.89	1.67	.64
29	1.49	1.33	1.12	2.53	.54
18	1.51	1.38	1.09	2.25	.72
11	1.67	1.36	1.23	3.12	.46
Mean		1.35±.02			
20	1.12	1.45	.77	1.18	.96
27	1.40	1.44	.97	1.37	.83
6	1.42	1.43	.99	2.29	.71
13	1.65	1.44	1.15	3.08	.46
Mean		1.44±.01			
26	1.10	1.61	.68	.98	1.08
25	1.17	1.60	.73	1.13	1.01
22	1.45	1.59	.91	1.23	1.00
36	1.49	1.61	.93	1.41	.93
9	1.52	1.61	.95	1.87	.94
Mean		1.60±.01			
33	1.09	1.72	.63	.86	1.21
2	1.12	1.74	.64	.94	1.10
31	1.30	1.74	.75	1.17	1.02
16	1.46	1.71	.85	1.89	.91
Mean		1.73±.02			
1	1.07	1.85	.58	.83	1.28
30	1.20	1.85	.65	1.01	1.05
35	1.20	1.83	.66	1.14	1.20
8	1.47	1.87	.83	1.34	1.20
Mean		1.85±.02			

material, obtained by dividing density scale by exposure scale, might correlate with ϕ_n. The data were then classified on the basis of equality of mean gradient. In Table XLIV each group contains positive materials for which ϕ_y is approximately constant. In the first group $\phi_y = .66 \pm .02$. It will be seen that ϕ_n varies from 1.05 to 1.20. While this variation is not extreme, still it represents several times the probable uncertainty applying to these values and there is no doubt that a print made from a negative for which ϕ_n is equal to 1.05 on material No. 35 would be considered by any observer as unsatisfactory. Conversely, print made

TABLE XLIV.

Classification of positive materials on the basis of equality in mean gradient, ϕ_y, illustrating the absence of correlation between the mean gradient of the optimal negative, ϕ_n, and the sensitometric characteristics.

No.	DS_y.	ES_y.	ϕ_y.	γ_y.	ϕ_n.
26	1.10	1.61	.68	.98	1.08
37	1.12	1.66	.67	.98	1.09
2	1.12	1.74	.64	.94	1.10
35	1.20	1.83	.66	1.14	1.20
30	1.20	1.85	.65	1.01	1.05
Mean			.66±.02		
20	1.12	1.45	.77	1.18	.96
14	1.18	1.51	.78	1.40	.92
32	1.33	1.68	.79	1.22	1.00
31	1.30	1.74	.75	1.17	1.02
4	1.34	1.79	.75	1.13	1.15
Mean			.77±.02		
3	1.17	1.20	.97	2.07	.57
6	1.42	1.43	.99	2.29	.71
27	1.40	1.44	.97	1.37	.83
28	1.46	1.56	.94	1.66	.79
9	1.52	1.61	.95	1.87	.94
36	1.49	1.61	.93	1.41	.93
Mean			.96±.03		
17	1.37	1.24	1.10	2.26	.60
7	1.37	1.29	1.06	2.62	.54
29	1.49	1.33	1.12	2.53	.54
18	1.51	1.38	1.09	2.25	.72
10	1.61	1.54	1.05	2.37	.66
Mean			1.08±.02		

from a negative for which ϕ_n is equal to 1.20 on material No. 30 would not give even passable quality. It is interesting to note that these two materials (No. 35 and No. 30) have identical values of density scale, exposure scale, and mean gradient differing only in gamma. It was thought, therefore, that perhaps some correlation between γ_y and ϕ_n might exist within a group of materials for which ϕ_y is constant. An effort was made to find this but without success, many apparently haphazard variations appearing in the computed values. We are forced to conclude, therefore, that the mean gradient of the positive material is of little value in predicting the gradient of the negative which will yield a print of optimal quality.

It might be expected, in the rendition of an object, having a fixed brightness scale, on positive materials of equal density scale, that the gradient of the optimal negative would be inversely

TABLE XLV.

Classification of positive materials on the basis of equality in density scale, DS_y. The ratio of ES_y to ϕ_n, shown in the last column, is not constant for materials of equal density scale, nor does this ratio correlate satisfactorily with any sensitometric factor.

No.	DS_y.	ES_y.	ϕ_y.	γ_y.	ϕ_n.	ES_y/ϕ_n.
20	1.12	1.45	.77	1.18	.96	1.51
26	1.10	1.61	.68	.98	1.08	1.40
37	1.12	1.66	.67	.98	1.09	1.43
33	1.09	1.72	.63	.86	1.21	1.42
2	1.12	1.74	.64	.94	1.10	1.58
1	1.07	1.85	.58	.83	1.28	1.44
Mean	1.10±.02					
3	1.17	1.20	.97	2.07	.57	2.10
15	1.17	1.32	.89	1.67	.64	2.06
14	1.18	1.51	.78	1.40	.92	1.64
25	1.17	1.60	.73	1.13	1.01	1.58
24	1.17	1.68	.70	.92	1.09	1.54
Mean	1.17±.01					
17	1.37	1.24	1.10	2.26	.60	2.07
7	1.37	1.29	1.06	2.62	.54	2.39
32	1.33	1.68	.79	1.22	1.00	1.68
5	1.36	1.69	.80	1.37	.99	1.68
4	1.34	1.79	.75	1.13	1.15	1.56
Mean	1.35±.02					
21	1.49	.93	1.60	3.21	.40	2.32
19	1.52	1.12	1.36	2.63	.47	2.38
29	1.49	1.33	1.12	2.53	.54	2.46
18	1.51	1.38	1.09	2.25	.72	1.89
28	1.46	1.56	.94	1.66	.79	1.97
9	1.52	1.61	.95	1.87	.94	1.71
36	1.49	1.61	.93	1.41	.93	1.73
34	1.46	1.73	.84	1.19	1.07	1.61
8	1.47	1.87	.83	1.34	1.20	1.56
Mean	1.49±.03					

proportional to the exposure scale of the material. If such a condition exists the ratio of exposure scale to negative gradient, ES_y/ϕ_n, should be constant. In Table XLV a classification has been made on the basis of equality in density scale. In the third group $DS_y = 1.35 \pm .02$. It will be noted that the values of ES_y/ϕ_n

vary over a wide range. There seems to be a tendency for this value to decrease as the contrast of the material decreases. It was thought, therefore, that by introducing some factor, such as Ω_y, γ_y, or ϕ_y, a definite correlation between the gradient of the optimal negative, ϕ_n, and some sensitometrically derived characteristic could be obtained. The values of ES_y/ϕ_n were therefore plotted as functions of various other factors. No systematic relationship was found. In the case of the third group of values shown in Table XLV the ratio of exposure scale to gradient of optimal negative plotted as a function of Ω_y did give a fairly good correlation, but in none of the other groups was the correlation even approximately satisfactory. The fact that the ratio ES_y/ϕ_n varies directly in a general way with the contrast of the positive material is in accord with the previous observation that as the contrast of the material increases a greater and greater proportion of the total density scale (or exposure scale) of the positive material must be sacrificed in order to obtain the optimal positive. This rather brief discussion will serve to illustrate the lack of any satisfactory correlation between the gradient of the optimal negatives as determined by the statistical method with the usual sensitometric characteristics of the positive material. The absence of such correlation is not surprising when the impossibility of utilizing the total available exposure scale of all materials for the best possible rendition of an object of fixed brightness scale is realized. If a portion of the available exposure scale of the positive material must be discarded it seems unreasonable to expect any correlation between the characteristic of the optimal negative and the characteristics of the positive material based upon the entire D-log E characteristic. Any correlation which exists must logically be expected between the characteristics of the optimal negative and characteristics of that portion of the D-log E curve actually used. Moveover it is apparent, from the excellence of the correlation existing between the factors of equations (20), (24), and (25), that the condition of vital importance in satisfactory reproduction is *limiting gradient* and its relation to the *maximum gradient* and *density scale* of the reproduction curve.

SUMMARY.

I. The general theory of tone reproduction has been reviewed and detailed application of the tone reproduction diagram to a specific case has been made. This shows graphically the quality

of reproduction obtained on positive material No. 9 by use of the negative which was found by statistical methods to result in optimal photographic quality.

II. The data suitable for use in the correlation work derived by statistical and sensitometric methods has been selected from the previous sections of the paper and tabulated in a form convenient for reference. These data may be classified in two groups. (*a*) The *sensitometric* constants of the positive materials are derived by *direct physical measurements* being obtained by computation from the coördinates of the four critical points of the *D*-log *E* characteristic. (*b*) The characteristics of the *optimal negatives, optimal positives,* and the *photographic reproduction* curves are dependent directly upon the *statistically* determined value of optimal negative gradient.

III. The method of computing the Bravais-Pearson correlation coefficient which is used as a measure of correlation precision was explained and illustrated by numerical examples. The coefficient obtained by using the *experimentally* observed values for two factors related in a known manner was found to be .95. The failure of this value to reach 1.00 must be attributed to the accumulated *experimental* and the *computational* errors. In the case of a relationship established by *statistical* methods a correlation coefficient of .95, therefore, is the maximum which can be expected in view of the experimental uncertainties involved. In order to compute the constants of the straight line fitting most perfectly the observed points, a least square method in which Σh^2 is minimum was used, h being the shortest distance between the observed point and the straight line. It is thought that this method is somewhat more satisfactory for application to this particular case than the usual method by which $\Sigma \Delta y^2$ is made a minimum.

IV. The *equalized limiting gradient* of the reproduction curve, $e\theta_r$, was plotted as a function of the *density scale* of the optimal positive, DS_p. This gave a distribution of points indicating that the function is of a straight-line type and that fairly good correlation exists. The Bravais-Pearson correlation coefficient was found to be .87 with a probable error of .04. Applying the least square method the equation of the straight line was found to be,

$$e\theta_r = .23\, DS_p + .036\,. \qquad (20)$$

The correlation coefficient is sufficiently high to indicate that the relationship holds with considerable certainty and hence its application to the prediction of negative gamma required to give optimal photographic quality on any positive material seems justifiable, at least for objects having a brightness scale equivalent to that of the object used in this work. A graphic method for finding *negative gradient* to fit a positive material for which the D-log E characteristic is known is developed and its application to a specific case illustrated.

Assuming that the constants in the statistical equation are independent of object brightness scale, BS_0, the application may be extended to include the reproduction of objects differing in brightness scale from the test-object used in this work. A graphic method for finding the *negative gradient* and *object brightness scale*, which will fulfill the requirements of statistical equation (20) when using a positive material for which the D-log E characteristic is known, has been developed and its application to a specific case illustrated.

V. By plotting the values of the *equalized limiting gradient of the reproduction* as a function of the *maximum gradient of the reproduction* it was found that the function relating these two factors was of the straight-line type. The correlation coefficient in this case was found to be .73 with a probable error of .035. Applying again the least square method, making Σa^2 minimum, the equation was found to be

$$e\theta_r = 1.24\,\gamma_r + .16. \qquad (24)$$

The correlation between these two values is not quite so perfect as that existing between $e\theta_r$ and DS_p. However, it is sufficiently good to establish considerable confidence in the validity of the relation between $e\theta_r$ and γ_r. The consequences of imposing the conditions required by equation (24) in addition to those of equation (20) have been considered, the results indicating that the two relationships are in general agreement and that there is a fairly wide range of object brightness scale which will meet the requirements of both.

VI. By plotting values of *equalized limiting gradient* of the *optimal positive*, $e\theta_p$, as a function of its *maximum gradient*, γ_p, a straight-line relationship is found to exist and the correlation

between the observed values is .95 with a probable error of .007. The equation, found by the method of least squares, is

$$e\theta_p = .22\,\gamma_p + .041\,. \qquad (25)$$

This correlation is extremely good, being equal to that existing between two factors for which, if there were no experimental errors, the correlation should be 1.0. This correlation therefore should hold with great precision and provides a direct means of determining just what portion of the total density scale of any positive material is available for the satisfactory reproduction of an object having a brightness scale of 1.44. The required negative gamma may then be computed directly. The extension of equation (25) to *objects of different brightness scale*, however, leads to unreasonable results and it seems probable that the constants in equation (25) must be dependent upon object brightness scale.

VII. The sensitometric data have been classified from the standpoint of (*a*) equality in exposure scale, ES_v, (*b*) equality of mean gradient, ϕ_v, and (*c*) equality in density scale, DS_v. These data have then been analyzed from the standpoint of the relation of optimal negative gradient, ϕ_n, to the various sensitometric constants. In no case was satisfactory correlation found. This absence of correlation is to be expected when it is considered that the proportion of the total exposure scale of the positive material which is actually used in producing the optimal positive varies appreciably, depending upon the contrast of the material. It seems reasonable, therefore, to expect correlation between the gradient of the optimal negatives and some characteristic relating to the utilized positive of the positive material rather than with values based upon the total scale of the positive material.

ERRATUM.

Section B, October, 1926, p. 483, Table VIII—last figure (1.79) should read 1.59.

The Fuchsin Sulphurous Acid Test.—HENRY LEFFMANN and MAX TRUMPER (*Bull. Wagner Free Inst. Science,* 1927, 11, 68–70) have made a study, in the Research Laboratory of that Institute, of the use of fuchsin sulphurous acid in the detection of methanol (methyl alcohol) in the presence of ethanol (ethyl alcohol) after selective oxidation of the former to formaldehyde. The fuchsin sulphurous acid reagent was prepared in three ways, according to: (1) The ninth decennial revision of the U. S. Pharmacopœia (1910), (2) the tenth decennial or current revision of that treatise (1920), and (3) the method of Fincke (*Zeitschrift für Untersuchung der Nahrungs- und Genussmittel,* 1914, 27, 246). All three solutions contain fuchsin in the same concentration (0.1 per cent.); but they differ with respect to the amount and the source of sulphurous acid, hydrochloric acid plus either sodium bisulphite or sodium sulphite. The solution prepared according to the ninth decennial revision of the U. S. Pharmacopœia was more satisfactory than that made according to the tenth decennial revision. The solution of Fincke gave either no color or only a transient color with acetaldehyde, and a permanent color with formaldehyde. When Fincke's solution was applied in the test for methanol, ethanol never interfered with the reaction, while glycerol gave a very blue color in the absence of both methanol and ethanol. This color would be mistaken for a positive test for methanol. Therefore, while Fincke's solution has some advantages, it, like the other solutions, cannot be used in testing alcoholic liquors, other than plain spirits, for the presence of methanol, unless a preliminary distillation is made. J. S. H.

New Method for Expression of Hydrogen-ion Concentration.—The hydrogen-ion concentration of a given solution is usually expressed as the logarithm of the reciprocal of its normality in that solution, and is designated by the symbol pH. EDGAR T. WHERRY (*Bull. Wagner Free Inst. Science,* 1927, 11, 59–64) has devised a simpler, arithmetical scale for the expression of hydrogen-ion concentration. The pH values 3.0, 4.0, 5.0, and 6.0 are designated active acidities of 10,000, 1000, 100, and 10, respectively. A neutral solution with a pH of 7.0 has both an active acidity and an active alkalinity of 0.0. The pH values 8.0, 9.0, 10.0, and 11.0 are designated active alkalinities of 10, 100, 1000, and 10,000, respectively. Descriptive terms are also used to further simplify the designation of reaction. When the active acidity is expressible in thousands, it is termed superacid, in hundreds mediacid, in tens subacid, and in units minimacid. Corresponding terms are used for active alkalinity. The regions of minimacidity and minimalkalinity are termed the zone of circumneutrality. Each descriptive term may be further qualified; thus, a superacid solution may have a high superacidity or a low superacidity, depending upon whether its pH is less than or greater than 0.5. J. S. H.

ULTRA-IONIZATION POTENTIALS OF MERCURY.

BY

ERNEST ORLANDO LAWRENCE, Ph.D.

National Research Fellow.

INTRODUCTION.

By bombarding mercury vapor with electrons of definite velocities—produced by magnetic analysis of the electrons from a thermionic source—it has recently been shown that there are a succession of distinct types of electron inelastic impacts in mercury vapor above the ionization potential having the following critical potentials: 10.6, 11.29, 11.70, and 12.06 volts, respectively.[1] Although a great many observations[2] in the past have been made on the ionization of mercury vapor by electron streams having thermal velocity distributions, there is no record of such ultra-ionization potentials. An estimate of the uncertainty introduced by a Maxwellian velocity distribution in the observation of ionization probabilities leads to the tentative conclusion that these critical potentials should have been observed. It was the purpose of this investigation to examine the apparent discrepancy existing between these recent experiments and the work of former investigators. It may be said at once that the experiment to be described shows that the ultra-ionization potentials are in evidence qualitatively when the vapor is bombarded with electron streams possessing Maxwellian velocity distributions, and, therefore, that the apparent discrepancy has been eliminated.

THE EXPERIMENT.

Fig. 1 shows the experimental arrangement. Electrons from an approximately equipotential region of an oxide-coated platinum filament were accelerated through an orifice A into a region where another accelerating field caused a portion to pass into an ionization chamber I. Positive ions produced by impacts of these electrons in the mercury vapor in the chamber were drawn to a grid G by a small field and to the wire collector W by a much larger field between G and W.

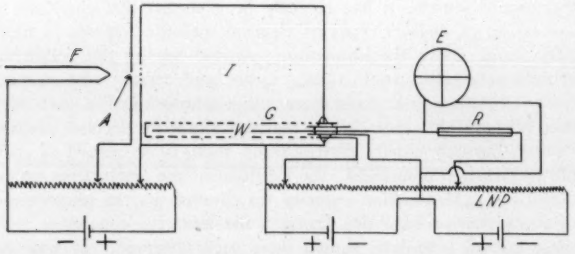
[1] Lawrence, *Phys. Rev.*, **28**, 947 (1926).

[2] See *National Research Council Bulletin* on "Critical Potentials," by K. T. Compton and F. L. Mohler for résumé of results.

The velocities of the electrons traversing the ionization chamber were varied by altering the difference of potential between the orifice A and the ionization chamber while the accelerating potential between the filament and A was kept constant (usually about 9 volts). In this way the current into the ionization chamber did not greatly increase when the velocities of the electrons were made greater.

The method of measurement of the positive-ion current was

FIG. 1.



somewhat unusual. The collecting electrode W was attached to the free quadrant of an electrometer E and the accumulation of a positive charge on the quadrant system was balanced by an equal and opposite current introduced by an attached India ink resistance R and a Leeds and Northrup potentiometer LNP, as is clearly shown in the diagram. In this way the electrometer served as a null instrument and the positive-ion currents were recorded as settings of the potentiometer.

The system was evacuated by diffusion pumps below values measurable on a McLeod gauge. However, undoubtedly wax vapors were present as well as the mercury vapor at room temperature.

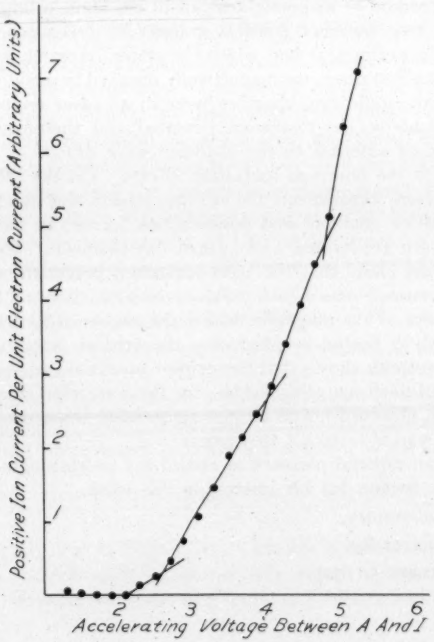
Helmholtz coils served to vary the magnetic field in the region of ionization.

RESULTS.

Typical experimental results are exhibited by Fig. 2. The ordinates record the observed ionization per unit electron current corresponding to electron accelerating voltages between A and the ionization chamber given by the abscissas. Several distinct

breaks in the curve are evident and if the most pronounced break is taken to correspond to a critical potential at 11.29 volts, the potentials at which ionization sets in, as well as the ultra-ionization potentials, are in agreement with the values 10.4, 10.6, 11.29,

FIG. 2.



11.7 volts, respectively. Within the experimental error these breaks are independent of the magnetic field in the region of ionization.

DISCUSSION.

Thus it has been demonstrated that former critical potential methods involving Maxwellian velocity distributions are capable of detecting in a qualitative manner ultra-ionization potentials in mercury and therefore that the apparent conflict in experimental

data is not real. That they have not been observed before perhaps is to be explained by the fact that the data were taken at greater intervals than 0.1 volt and with much larger currents than the electron currents of about 10^{-9} ampere used in the present investigation.

The method of magnetic analysis of the bombarding electrons involved two inherent possible causes for erroneous interpretation of experimental data which the present research has cleared up. In the first place, the magnetically resolved beams of electrons entering the ionization chamber were in all cases retarded from velocities above the ionization potential and therefore positive ions formed external to the chamber were drawn in and contributed to the observed ionization effects. On the other hand, in the present experiments the electron stream was accelerated to the ionization chamber and positive ions formed in the external region were accelerated away from the chamber. Thus, these experiments show that the ultra-ionization potentials are not a peculiar consequence of such diffusion of positive ions. Secondly, the presence of the magnetic field in the region of ionization was conceivably a factor in producing the critical potentials. The present research shows that the critical potentials are quite independent of such magnetic fields—for the ionization curve breaks have been observed not to change appreciably when the magnetic field was varied from 0.5 to 3 gauss.

It is an especial pleasure to record my indebtedness to Prof. W. F. G. Swann for his interest in this work.

Sloane Laboratory,
Yale University,
February 5, 1927.

RELATIONSHIP BETWEEN UNIT ELECTROLYTIC CONDUCTIVITY AND FARADAY'S LAW.

BY

MARION EPPLEY, Ph.D.

The Eppley Laboratory; Member of the Institute.

FARADAY'S Law is the most general statement of the relationship governing the passage of electricity through a solution of an electrolyte. Merely to state that to deposit one gram-equivalent of any element or radicle from its solution there must pass 96,500 coulombs of electricity, is to leave electromotive force, resistance (and the factors determining it such as concentration, temperature, electrode-area and electrode-distance) and time (and hence current-strength) to be determined by the particular conditions of each case. By fixing the values of certain of these factors, the connection between Faraday's Law and unit electrolytic conductivity can be established.

The unit of electrolytic conductivity is defined as the conductivity (reciprocal of the resistance) of a column of liquid one centimetre in length and one square centimetre in cross-section, the resistance of which is one ohm.[1] Electrolytic conductivity, k, therefore is expressed in terms of resistance by the equation

$$k = \frac{1}{R}.$$

This is equivalent to Ohm's Law when the electromotive force is unity, so that k must be numerically equal to the current in amperes between electrodes one centimetre square and one centimetre apart when the electromotive force is one volt.[2]

Hence, if the conductivity of a liquid be unity, a current of one ampere will flow—that is, one coulomb of electricity per second will be transferred—between the opposite faces of the cube defined above, the electromotive force being one volt. This coulomb of electricity will deposit a definite quantity of matter.

[1] Tower, "The Conductivity of Liquids," p. 1, Chem. Pub. Co., 1905.

[2] Lewis, "A System of Physical Chemistry," 1, p. 190, Longmans, 1918.

The mass in grams of an element or radicle deposited by one coulomb is the electrochemical equivalent of that element or radicle.[3] In this case, by definition of the ampere, the time of deposition of an electrochemical equivalent will be one second.

Therefore, when the conductivity of a solution is unity, a current of one ampere (one coulomb per second) passed through it between electrodes one centimetre square and one centimetre apart will deposit upon these electrodes one electrochemical equivalent per second.

This expresses the relationship between Faraday's Law and unit electrolytic conductivity.

[3] Hering and Getman, "Electrochemical Equivalents," p. 65, Van Nostrand, 1917.

MEDAL DAY MEETING.

THE annual Medal Day Meeting of The Franklin Institute, which was also the regular monthly meeting for May, was called to order by the President, Dr. Wm. C. L. Eglin.

The President announced that the minutes for the last meeting had been printed in full in the May number of the JOURNAL and that, if there were no objection, the minutes would stand as printed. No objection being offered, the minutes were approved.

The President then called upon the Secretary, Dr. Howard McClenahan, who stated that he had two announcements to make, of especial interest to the members. He reported: "Mr. President, I wish to make two announcements which should be of interest to the members of the Institute. In the first place, Prof. W. F. G. Swann, head of the Department of Physics, Yale University, has been invited to become the head of the Bartol Research Foundation and has accepted that invitation. Professor Swann will take up his active work in connection with our Laboratories on September first, next. It is needless to say that his appointment brings with it promise of very great achievement to the Laboratories, because Professor Swann is one of the leading physicists and research men of the United States."

This announcement was received with enthusiastic applause, and the Institute was congratulated upon its good fortune by the other scientists present.

The Secretary continued: "The other matter of which I should like to make mention is one that has been noted recently in the daily papers—the fact that the Institute hopes to build upon the Parkway a great museum of science, technology, and industry, which shall be a monument to the industrial accomplishments of the business men and the scientists of Philadelphia, and a most effective addition to the agencies for the teaching of science throughout this city. The Museum, if the City Council will look upon our request favorably, will be located on the Parkway, between Eighteenth and Nineteenth Streets, will be in its purpose chiefly educational, and will be, we firmly believe, a very distinguished monument to science and its accomplishments in Philadelphia, and an instrument which will bring great credit to this

institution. As our plans develop, the members will be fully informed of what we are doing and of the things which we hope to accomplish."

The President then announced that the awarding of Medals was the next order of business, and stated that all the medals which have been recommended during the present Institute year by the Committee on Science and the Arts would be presented at this time.

Award of Longstreth Medals.

To Mr. Wilfred Lewis, Tabor Manufacturing Company.

The Chair recognized Mr. Coleman Sellers, III, as sponsor for Mr. Lewis. Mr. Sellers said: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, Mr. Wilfred Lewis, of the Tabor Manufacturing Company, as the recipient of the Edward Longstreth Medal, 'in consideration of the novelty and ingenuity of the design and the successful operation of his Shockless Jarring Machine.'"

In presenting the medal, certificate and report, the President said: "Mr. Lewis, this is the second time the Institute has had the pleasure of honoring you. Upon the recommendation of its Committee on Science and the Arts, and the unanimous approval of its Board of Managers, it gives me much pleasure, by virtue of the authority conferred upon me, to present this medal and certificate to you."

In acknowledging the award, Mr. Lewis spoke as follows: "Mr. President, Ladies and Gentlemen: Some two thousand years ago, Archimedes, the greatest mathematician and mechanical genius of his time, longed for a place whereon to stand that he might, with his lever, move the world. But he never found the right place, and it does not appear that he ever developed a suitable lever for this purpose. His aspiration, however, gripped the minds of men, and so added to his fame that it may truly be said he did move the world in spirit for many centuries, until a riper intelligence came to find that a place to stand was not needed so much as a place to fall, because the earth was sure to gravitate toward anything falling on its surface, and move through a distance inversely proportional to its relative mass. Not very much, to be sure, because the mass of our little planet is so great that a

steel ball two thousand feet in diameter, weighing a billion tons and falling a mile, would not move the earth, as a whole, one-millionth part of an inch, yet it cannot be doubted that the impact from such a falling mass would set up local disturbances in the crust of the earth of a very destructive character.

"But instead of dropping a billion tons a mile, I found that one ton falling an inch or two was quite enough to set up vibrations in the grounds that could be felt for a considerable distance and were capable of doing more or less damage to foundations or finished molds on foundry floors.

"These considerations led to the shockless jarring machine for which The Franklin Institute has so graciously awarded me the Edward Longstreth Medal.

"It is in effect like cracking two stones together in midair and it has been described as a rising anvil to meet a falling table, but there is really a good deal more to it, because the table and anvil must keep in step with each other, and, if time permitted, I might go on to show how the desired result has been accomplished under a great variety of conditions. But there are other matters of more general interest on the program and I shall not detain you any longer than to express my appreciation and thanks for the honor conferred upon me."

To Mr. Harry W. Hardinge, Hardinge Company, York, Penna.

In connection with this award, the Secretary announced that the recommendation of the Committee on Science and the Arts had been adopted too late to permit Mr. Hardinge's name to be printed on the programme.

The Chair recognized Mr. F. V. Hetzel, sponsor for Mr. Hardinge, who was represented by his son, Harlowe Hardinge. Mr. Hetzel said: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, Mr. Harlowe Hardinge, representative of Mr. Harry W. Hardinge, who will accept the Edward Longstreth Medal awarded to Mr. Hardinge, 'in consideration of the ingenuity of his Rotary Air Classifier and of its usefulness in increasing the capacity of Hardinge Mills and other mills of that type, and of permitting a closer and more uniform separation of the product.'"

The President then presented the medal, certificate and report, as follows: "Mr. Hardinge, upon recommendation of the Institute's Committee on Science and the Arts, and the unanimous approval of the Board of Managers, I have the honor to present to you the Edward Longstreth Medal and Certificate."

In accepting the award, Mr. Hardinge said: "On behalf of my father, Mr. Harry W. Hardinge, I wish to thank The Franklin Institute for the honor conferred upon him, as I know he deeply appreciates it."

To Mr. James F. Smathers, Smathers Power Typewriting Company.

The Chair recognized Mr. C. W. Bates, sponsor for Mr. Smathers. Mr. Bates: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, Mr. James F. Smathers, of the Smathers Power Typewriting Company, as the recipient of an Edward Longstreth Medal, 'in consideration of the ingenuity on his part as the original inventor of an electrical drive as applied to typewriters.'"

The President made the award in the following terms: "Mr. Smathers, it is my pleasure to present to you the Edward Longstreth Medal, upon recommendation of the Committee on Science and the Arts and the unanimous approval of the Board of Managers of The Franklin Institute."

Mr. Smathers said: "I appreciate this honor and thank you for the medal."

Award of Wetherill Medal.

To North East Appliances Inc., with special mention of Mr. Russell G. Thompson.

The Chair recognized Mr. Bates, sponsor for the North East Appliances Inc. Mr. Bates: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, the representative of the North East Appliances Inc., of Rochester, N. Y., as the recipient of the John Price Wetherill Medal, 'in consideration of the resourcefulness and painstaking care necessary for the successful mechanical and commercial development of such a highly complicated piece of apparatus as their Power Drive for Type-

writers. Mr. R. G. Thompson, the representative, receives special mention in this award, because of the fact that he, above all others, has been responsible for the successful development of the power drive for typewriters.' "

The award was then made to Mr. Thompson by the President: "Mr. Thompson, upon the recommendation of our Committee on Science and the Arts, I have the pleasure in presenting to you as the representative of the North East Appliances Inc., the John Price Wetherill Medal and the accompanying certificate, with special reference to your work in this award."

Mr. Thompson accepted the award as follows: "On behalf of our Company, I wish to express our sincere appreciation of this award. We hope that the benefits which will result to the public will justify the confidence in us as expressed in this award."

Award of Levy Medal.

To Dr. William David Coolidge, General Electric Company.

The Chair recognized Mr. Benjamin Franklin, sponsor for Doctor Coolidge. Mr. Franklin: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, Dr. William David Coolidge, of the General Electric Company, as the recipient of the Louis Edward Levy Medal, 'for his paper entitled "The Production of High-voltage Cathode Rays Outside of the Generating Tube," published in the JOURNAL OF THE FRANKLIN INSTITUTE for December, 1926.' "

The President said: "Doctor Coolidge, this is the second time that the Institute has had the privilege of recognizing your work in physics. On this occasion we are particularly happy to acknowledge that through the JOURNAL OF THE FRANKLIN INSTITUTE you gave to the world your work on cathode rays outside the generating tube. This is probably the most important advancement in physics made during the past year. It is an especial pleasure to present to you the Louis Edward Levy Medal, which is done upon recommendation of our Committee on Science and the Arts, with the unanimous approval of our Board of Managers."

Doctor Coolidge, in acknowledging the award, said: "I wish to express to you and to the members of The Franklin Institute my very deep appreciation of this honor."

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Award of Potts Medals.

To Prof. George Erle Beggs, Princeton University.

The Chair recognized Mr. Benjamin Franklin, sponsor for Professor Beggs. Mr. Franklin: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, Prof. George Erle Beggs, of Princeton University, as the recipient of the Howard N. Potts Medal, 'in consideration of the development of a superior method of determining stresses in complex structures embodying the application of fundamental principles.'"

The President, in making the award, said: "Mr. Beggs, we of The Franklin Institute are familiar with your work on account of the paper presented by you before the Institute, and we are particularly happy, upon the recommendation of our Committee on Science and the Arts, to award to you the medal which I now have the honor of presenting to you."

Professor Beggs acknowledged the medal as follows: "It is with great pleasure that I accept this medal. You have my sincere thanks. I wish to call the attention of the audience to the fact that the principle used by me is a very old one, having been used by Maxwell. This but adds to the credit of one of our scientists, who has been honored by us all for many years."

To Commander Marion Eppley, Eppley Scientific Laboratories, Newport, R. I.

The Chair recognized Mr. C. A. Hall, sponsor for Commander Eppley. Mr. Hall: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, Dr. Marion Eppley, of Newport, R. I., as the recipient of the Howard N. Potts Medal, 'in consideration of the origination and improvements of processes in the manufacture of standard cells and the development of a technique which ensures their successful quantity production.'"

In presenting the medal, the President said: "Commander Eppley, we have had the pleasure and privilege of having a paper presented by you on standard cells. We are particularly happy, therefore, that the Committee on Science and the Arts has recommended the award to you of the medal and certificate which I have the honor of presenting to you."

FRANKLIN INSTITUTE LIBRARY

Commander Eppley said: "I wish to thank you on my own behalf, and on behalf of my co-workers, for this very great honor which you have bestowed upon us. This medal shall always be an inspiration to us."

Award of Cresson Medals.

To Dr. Edward Leamington Nichols, Professor Emeritus of Physics, Cornell University.

The Chair recognized Prof. W. R. Wright, sponsor for Doctor Nichols. Professor Wright: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, Prof. E. L. Nichols, Professor Emeritus of Physics, Cornell University, as the recipient of the Elliott Cresson Medal, 'in consideration of his achievements as an investigator and, in particular, of his services to the development of the science of physics in this country.'"

The President presented the medal as follows: "Professor Nichols, no words of mine are needed to describe your position in science, but we of the Institute are particularly happy that our Committee on Science and the Arts has awarded to you the Elliott Cresson Gold Medal and Certificate, which I now have the honor of handing to you."

Professor Nichols expressed his gratitude as follows: "It is beyond my power to express the depth of my feeling upon receiving this honor, and particularly from The Franklin Institute, which I deem a double honor, because the Institute combines recognition of the useful as well as of the pure science."

To Dr. Dayton Clarence Miller, Research Professor of Physics, Case School of Applied Science.

The Chair recognized Prof. T. D. Cope, sponsor for Professor Miller. Professor Cope: "Mr. President, I have the honor to present to you, upon recommendation of the Committee on Science and the Arts of The Franklin Institute, Dr. Dayton Clarence Miller, Research Professor of Physics, Case School of Applied Science, as the recipient of the Elliott Cresson Medal, 'in consideration of his fundamental investigations in acoustics and his outstanding contributions to that branch of physical science.'"

The President said: "Professor Miller, upon recommendation

of the Institute's Committee on Science and the Arts, and the unanimous approval of the Board of Managers, I have the honor to present to you the Elliott Cresson Gold Medal."

Professor Miller acknowledged the award: "I wish to express my profound appreciation of this very great honor. I thank you."

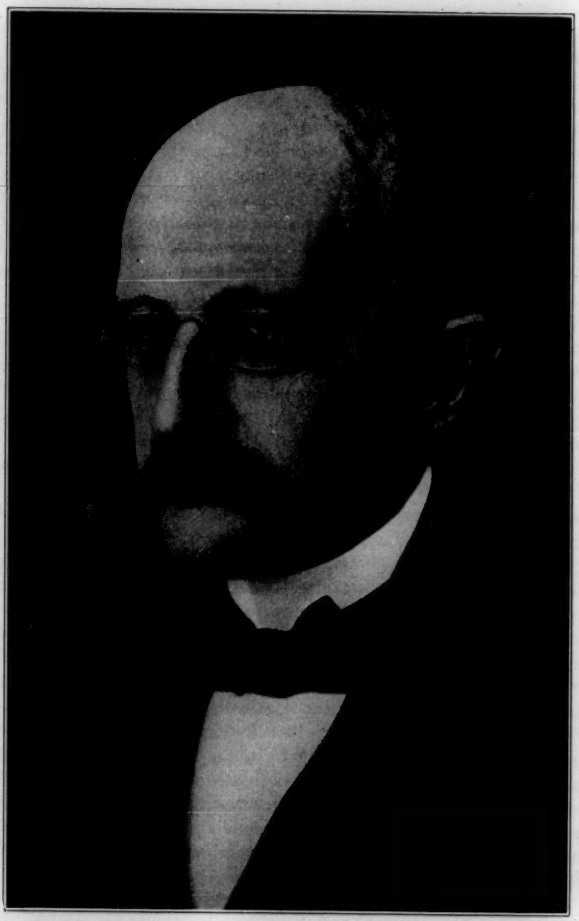
Award of Franklin Medals.

To Prof. Max Planck, Ph.D., University of Berlin, represented by the Counselor of the German Embassy, Dr. O. E. Kiep.

The Chair recognized Dr. James Barnes, sponsor for Professor Planck. Professor Barnes: "Mr. President: The Board of Managers of The Franklin Institute on the recommendation of its Committee on Science and the Arts was unanimously of the opinion that a Franklin Medal be awarded to Dr. Max Planck, Professor of Mathematical Physics and Director of the Institute of Theoretical Physics of Berlin, 'in recognition of his valuable contributions to the field of thermodynamics, particularly for his law of radiation, in the development of which he employed, for the first time, the revolutionary idea of the fundamental indivisible quantity of radiant energy called the "quantum."'

"Max Planck was born at Kiel April 23, 1858. He studied at the Universities of Munich and Berlin and in 1879 he received the degree of Ph.D. from Munich. There he remained as Privat Docent till 1885, when he was appointed to the Professorship of Physics at Kiel. In 1889 he became Professor of Mathematical Physics and Director of the Institute of Theoretical Physics at Berlin. In 1913–1914 he was Rector of the University of Berlin and in 1920 he received the Nobel Prize for his distinguished physical investigations.

"Doctor Planck has devoted himself to the study of theoretical physics and in particular to that of thermodynamics. He gained international fame by his law of radiation, proposed in 1901, which asserts that radiation is emitted and absorbed in integral multiples of a certain indivisible quantity of energy which depends only on the frequency of the radiation. In several fields of physical investigations his theory has been confirmed with the most surprising accuracy, especially in the study of the specific heats of solids, in the kinetic theory of gases, in spectroscopy and photo-electricity.



MAX PLANCK, PH.D.

"He has published numerous papers and essays and many books.

"Mr. President, it is a source of regret to us that, because of engagements of long standing, Doctor Planck cannot be with us to receive this medal in person. We are, however, highly appreciative of the courtesy of Doctor Kiep, Counselor of the German Embassy, in coming to us from Washington to receive, on behalf of Doctor Planck, this medal. I, therefore, have, Sir, the honor and pleasure of presenting to you Doctor Kiep who, on behalf of the German Government, will accept the Franklin Medal and Certificate for Dr. Max Planck, and will see that they are transmitted to him."

The President then presented the award to Doctor Kiep: "Doctor Kiep, I am happy to announce that by virtue of the authority conferred upon me and upon recommendation of the Committee on Science and the Arts and the unanimous approval of the Board of Managers of The Franklin Institute, there has been awarded to Dr. Max Planck a Certificate and Franklin Medal. This also carries with it Honorary Membership in the Institute. The Franklin Medal is the highest honor, and Honorary Membership is next. I would ask you to convey to Doctor Planck this Medal and Certificate and Certificate of Honorary Membership."

Doctor Kiep, in acknowledging the award on behalf of Doctor Planck, said: "It is a great honor for me to represent the recipient of the exceptional distinction which has just been conferred. On behalf of Prof. Max Planck, of Berlin, I beg to express sincerest thanks to the President of the Board of Managers of The Franklin Institute for the Franklin Medal and the Honorary Membership of the Institute, which have been conferred upon him. Professor Planck deeply regrets that he has been unable to come over from Germany and be present here at this memorable meeting, but hopes that he may find occasion next year, when he expects to deliver lectures in America as exchange professor, to express to you personally his high appreciation and his sincerest gratitude for the great honor you have extended to him.

"I am not in a position to represent Professor Planck in the language and the thoughts of the science he stands for, and in his achievements which have been rewarded by you to-day; but I understand that a scientific paper prepared by him will be read

by one of his American colleagues at this meeting. What I may do, however, is to express to you his personal feelings with reference to this auspicious occasion.

"I have the honor to know Professor Planck personally and admire equally his achievements in the domain of science and his character and personality. Were he here to-day, he would modestly reject the praise extended to his work and his merits, and would ask to be permitted to accept the high distinction conferred upon him as a tribute to the branch of science he represents, and to the scientific work of the country to which he belongs. And if there is something that appeals to me particularly, as a representative of diplomacy and international politics, it is the fact that the action you have taken in presenting the Franklin Medal to-day, extends to a scientist of a foreign country, thus demonstrating that the field of science is without national frontiers or barriers, and that the spirit of scientific work is universally imbued with the idea of the coöperation of all nations.

"At no time, I believe, has the exercise and development of the spirit of international coöperation been more urgently requisite than to-day, and I have the impression that the desire for international coöperation and understanding has been expressed and acted upon by no country so extensively as by the United States. This refers equally to the domain of politics as to economic and other spheres of the work of civilization.

"German scientific and research work has, in the years of economic distress brought about by inflation and post-war calamities, received great assistance from the philanthropic institutions of your country. To express gratitude for this wonderful assistance, I feel no more suitable occasion could be found than this meeting where a representative of German science has been honored by the greatest distinction your country has to confer in the domain of physical science which you have just conferred. And in voicing these thanks, I think I should like to add that it is the sincere desire of German science as represented by Max Planck to collaborate as closely and as intimately as possible with American science in the mutual interest of those concerned and for the benefit of all humanity."

The Secretary then announced that Professor Planck has sent a message which he reported as follows: "I received yesterday a cablegram from Professor Planck which my best German enables

me to translate as follows: Doctor Planck wishes us to know that he is present with us in spirit to-day and that he is filled with appreciation of the honor which we have attempted to pay to him. It seems to me that this message, coming in this way, is but an added proof of the delightful personality of the scholar whom we are glad to honor to-day."

To Dr. George Ellery Hale, Honorary Director, Mount Wilson Observatory. Represented by Prof. Harlow Shapley, Harvard University.

The Chair recognized Dr. James Barnes, sponsor for Doctor Hale. Doctor Barnes: "Again Mr. President: The Board of Managers of The Franklin Institute on the advice of the Committee on Science and the Arts was unanimously of the opinion that a Franklin Medal be awarded to Dr. George Ellery Hale, Honorary Director of the Mount Wilson Observatory, of Pasadena, Calif., 'in recognition of his numerous and highly important discoveries and researches in solar physics and of his basic inventions for the investigation of solar phenomena, of his eminent success in organizing and directing astronomical observatories and of his establishment and promotion of plans and means for coördination and advancement among the sciences.'

"George Ellery Hale was born in Chicago on June 29, 1868. He was graduated from the Massachusetts Institute of Technology in 1890 and was at once appointed Director of the Kenwood Observatory in Chicago. In 1897 he was appointed Professor of Astrophysics at the University of Chicago. From 1892 to 1895 he was Editor of *Astronomy and Astrophysics* and is now one of the editors of the *Astrophysical Journal*. In 1895 he became Director of the Yerkes Observatory and in 1904 he was appointed the Director of the Mount Wilson Observatory of the Carnegie Institution.

"The formation of the National Research Council was largely due to Doctor Hale's efforts, he is now its Honorary Chairman.

"Doctor Hale's outstanding scientific accomplishment is the invention of the spectroheliograph, an instrument whereby different layers of the solar atmosphere (called the chromosphere) are photographed by the different lines emitted by incandescent hydrogen and calcium. His outstanding discovery is the obser-

GEORGE ELLERY HALE, Sc.D., PH.D., LL.D.

vation of the Zeeman effect in sun-spots and the determination of the magnitude of their magnetic fields.

"Doctor Hale has been the recipient of medals from numerous scientific societies and of honorary degrees from many universities in this country and abroad.

"He is the author of a very large list of contributions to the astronomical and astrophysical journals and in addition he has written several books.

"Mr. President, it is a source of keen regret to all that, because of illness, Doctor Hale is unable to be here to-day. However, a colleague in astronomy, himself a brilliant student of astronomical physics, has been kind enough to come from Cambridge, Mass., to accept the medal on behalf of Doctor Hale and to read for us a paper which Doctor Hale has prepared. I therefore have much pleasure, Sir, in presenting to you Prof. Harlow Shapley, Director of the Observatory and Professor of Astronomy in Harvard University, who will receive on behalf of Dr. George Ellery Hale, the Franklin Medal and Certificate."

The President then presented the award to Professor Shapley: "Doctor Shapley, by virtue of the authority conferred upon me, I am happy to announce that with the unanimous approval of the Board of Managers and the Membership of The Franklin Institute, upon the recommendation of its Committee on Science and the Arts, there has been awarded to Doctor Hale the Franklin Medal and Certificate, the highest award within the gift of the Institute. Also, Doctor Hale has been elected to Honorary Membership. It is my pleasure to present these to you, with my best wishes for his speedy recovery and with the hope that he may be spared long to enjoy the good things of this earth which he so richly deserves."

The Secretary made the following announcement: "We received to-day a message from the International Education Board of New York City. This Board is doing more, probably, than any other one group to remove the ravages of war from the scientific societies and laboratories of the countries of Europe. They are deeply interested in what we are doing here to-day and have sent a message of good will expressing high regard for Doctor Planck and Doctor Hale."

The President: "As already announced, a paper has been prepared by Professor Planck to be presented by Professor Cope.

The Institute is indebted to Doctor Cope for translating this paper. It is interesting to note that Professor Cope is a former pupil of Doctor Planck." Professor Cope then read the paper on "The Physical Reality of Light-quanta," by Prof. Max Planck (p. 13).

The President: "We wish to express the thanks of the Institute for the presentation of the paper."

The President: "We regret exceedingly, of course, that Doctor Hale is not here. We are, however, very fortunate in having with us Doctor Shapley. It is my pleasure to present Doctor Shapley, who will read a paper by Doctor Hale, prepared especially for this occasion."

Doctor Shapley: "I had hoped that Doctor Hale's paper would be one so highly difficult and technical that I would not dare to read it to you. That, unfortunately for me, does not appear to be the case. I dislike to read a paper for three reasons: One, I have only done it once before. I will never forget that time—and neither will my audience; two, it is well to keep your eye on your audience and this you cannot do when you are reading; three, I don't see how anyone can stand still for fifteen minutes. If some one will invent a machine which will enable anyone to stand still for that length of time, that inventor will richly deserve a Franklin Medal." Professor Shapley then read a paper on "The Sun as a Research Laboratory," by Dr. George Ellery Hale (p. 19).

The President: "Doctor Shapley, I desire to express the thanks of The Franklin Institute to you for coming to us and presenting Doctor Hale's paper.

"I wish also to express our thanks to the members of the Institute who have assisted us in the exercises of this afternoon.

"As there is no further business, the meeting is adjourned."

Federal Caustic Poison Act.—"An Act to safeguard the distribution and sale of certain dangerous caustic or corrosive acids, alkalies, and other substances in interstate and foreign commerce" was approved by the President on March 4, 1927 (Public Document No. 783, 69th Congress, 6 pages). This Act may be cited as the "Federal Caustic Poison Act." The term "dangerous caustic or corrosive substance" is applied to hydrochloric acid, sulphuric acid, nitric acid, carbolic acid (phenol), oxalic acid, oxalates, acetic acid, free or combined hypochlorous acid, potassium hydroxide, sodium hydroxide, silver nitrate, ammonia water, and certain of their preparations including those containing 20 per cent. or more of free or chemically unneutralized acetic acid; 10 per cent. or more of oxalates, or of free or chemically unneutralized hydrochloric acid, sulphuric acid, oxalic acid, potassium hydroxide or sodium hydroxide; 5 per cent. or more of silver nitrate, phenol, or free or chemically unneutralized nitric acid or ammonia, and preparations (other than calx chlorinata, bleaching powder, and chloride of lime) yielding 10 per cent. or more by weight of available chlorine. The retail parcel, package, or container of a dangerous caustic or corrosive substance is misbranded unless it bear a conspicuous, easily legible label containing the common name of the substance, the name and place of business of the manufacturer, packer, seller, or distributor; the word "poison," and, if intended for household use, directions for treatment in case of accidental personal injury by the substance. Misbranded parcels, packages and containers of these substances are denied shipment in interstate or foreign commerce. However, exports may be branded in accordance with the laws of the foreign country. Provision is made for the exclusion of misbranded imports; and penalties are provided for violations of the Act. The enforcement of the Act is placed in the hands of the Secretary of Agriculture. The Act became effective upon its passage, but the penalty and condemnation clauses concerning violations are effective six months thereafter.

J. S. H.

Biochemistry of Manganese.—The researches of ROBERT F. MCCRACKAN and EMANUEL PASSAMANECK, of the Medical College of Virginia (*Archives Path. and Lab. Med.*, 1926, 1, 585–587), show that manganese, which gains entrance to the human body, may be excreted through the kidney. They studied five persons who worked in a manganese grinding plant. Three of these persons had a definite manganuria; one of them showed 5 milligrams of manganese per litre of urine, the other two smaller amounts. The observation was made that a definite pink color is imparted to an aqueous solution containing 0.2 milligram of manganese as permanganic acid per litre, *i.e.*, by 0.2 part of manganese per million.

J. S. H.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

PROTECTING AIRCRAFT AGAINST CORROSION.

One of the greatest drawbacks in constructing airplanes and airships from the light, strong, aluminum alloy, duralumin, is the fact that on exposure, especially to salt air, the alloy, particularly when in sheet or other thin forms, may become brittle through a type of corrosion which is not apparent on the surface and which therefore cannot be detected upon inspection. This sort of corrosion weakens the bond between the crystal grains that make up the alloy and so is termed "intercrystalline embrittlement." The tiny, submicroscopic particles, which give the alloy its desirable properties, tend to collect at the grain boundaries. If this collection at the boundaries can be avoided, the tendency toward embrittlement is reduced.

In commercial practice, the alloy is generally heated in a bath of fused salts (nitrates) and quenched for heat-treatment in boiling water; boiling water being used because the adhering salt is more readily dissolved than in cold water. On the other hand, the greater speed of quenching in cold water minimizes the time for collection of the hard particles at the grain boundaries and hence markedly increases the resistance to intercrystalline corrosion.

Work of the past two years at the bureau in coöperation with the Navy, Army, and National Advisory Committee for Aeronautics has shown that, in laboratory corrosion tests, duralumin quenched in cold water is markedly superior to that quenched in hot water or oil, but even this does not fully solve the problem, and protection of the surface of the duralumin is also advisable.

Many types of protective coatings have been tried, several of which appear quite effective. Few coatings of the paint or varnish type, however, are completely impervious to moisture and most of them tend to crack or blister with age, thus losing their adhesion and some of their protective power. However, commercially pure aluminum is not subject to intercrystalline embrittlement because it does not contain the hardening particles that tend to create a pathway for the intercrystalline attack.

Hence, if an adherent coating of unalloyed aluminum be pro-

* Communicated by the Director.

vided over the duralumin, good protection is afforded. This idea, evolved at the bureau some three years ago, was tried out in laboratory tests. Duralumin, so protected, proved superior to that protected in other ways because the aluminum coating will stand more scratching without baring the duralumin. Specimens coated with aluminum by the metal-spray process have been exposed to salt air for a year without deterioration.

Reports of the good behavior of aluminum-coated duralumin have been made to the coöperating agencies, have been briefly commented on before,[1] and the information has been available to producers of duralumin. It is gratifying to note that one of the chief producers of duralumin has, by independent laboratory tests, corroborated the bureau's experience as to the good performance of aluminum-coated duralumin in resistance to corrosion and is just starting to make commercially available, duralumin in sheets coated with aluminum by a process which the manufacturer has worked out but which has not yet been described in detail. The product is of the same general type, however, as that which the bureau has tested extensively.

It therefore appears that by cold water quenching, by the use of aluminum coatings, and by the use of various protective films, varnishes, grease coatings, etc., as a further insurance against corrosion, the greatest drawback to long life and safety of duralumin for aircraft construction may be almost completely eliminated, and by means that are commercially practical.

EFFECT OF RELATIVE HUMIDITY AND TEMPERATURE ON THE TENSILE PROPERTIES OF VULCANIZED-RUBBER COMPOUNDS.

THE coöperative work between the bureau and the American Chemical Society in studying the effect of relative humidity and temperature on the tensile properties of vulcanized rubber was announced in *Technical News Bulletin*, No. 118. One phase of the problem has been completed. The study of five compounds, each at five states of cure, was made at five temperatures ranging from 5 to 45° C., and at six humidities ranging from 0 to 100 per cent. relative humidity.

The results show that the temperature of the testing room should be controlled within one or two degrees, and that relative

[1] *Forging, Stamping, Heat Treating*, 12, p. 373, Oct., 1926.

humidity also should be controlled to some extent. It is probable that a control of relative humidity within a range of 10 per cent. (plus or minus 5 per cent.) would be satisfactory.

There is no evidence that state of cure influences to any great extent the effect of either temperature or relative humidity. Gum stocks seem to be affected slightly more than heavily compounded stocks. The effect of relative humidity and temperature was not influenced by the type of accelerator or the softener used in the compound.

The first phase of the work consisted in a study of the effect of humidity and temperature after vulcanization and during testing. The compounds were conditioned two and six days. Two days represent the usual time in commercial practice, while the six-day exposures were made to get nearer the maximum effect.

The next phase of the work is to determine the effect of humidity and temperature upon the unvulcanized compounds. The compounds will be mixed at the bureau and carefully conditioned over ranges of relative humidity and temperature before vulcanization. After vulcanization the specimens will receive uniform conditioning as to temperature and humidity to determine the effect of the varying conditions previous to vulcanization.

The results in detail will be reported to the rubber section of the American Chemical Society when the second phase of the work has been completed.

MANILA ROPE WASTE FOR PAPER MANUFACTURE.

PAPER-MAKING tests have recently been made at the Bureau of Standards to determine the suitability of manila rope waste for paper manufacture. The waste employed in the experimental work was refuse fibre from the manufacture of manila cordage at the rope walk of the Boston Navy Yard. At present, little, if any, of the waste derived from manila hemp at cordage mills is used commercially, most of it being burned under the boilers as fuel. The experimental tests were made with a view to finding a profitable use for the refuse fibre of the cordage industry.

Manila hemp (*Musa textilis*) is obtained from the leaf stalks of a plant of the banana family and is one of the principal products of the Philippine Islands. The fibre is tough and strong and is used primarily for textiles and cordage. The material used in

the paper industry is obtained almost entirely in the form of old rope, and the papers produced from it are commonly called "rope" papers. Rope papers are characterized by great strength, toughness, and wearing qualities, and are ordinarily used for shipping tags, wrappers, bagging, etc., in which strength is essential.

The pulping and paper-making operations employed in the experimental tests on the waste were those commonly used in paper manufacture. Soda ash was the digesting agent that gave best results. Tests on kraft and sulphite wood pulps and on old rope stock were included for comparison. A good yield of paper-making was obtained from the waste and the paper produced was of sufficient strength for a variety of uses. Measurements on the finished paper indicate that manila rope waste does not compare favorably with old rope stock but is considerably better than sulphite wood pulp.

A complete report of this work was published in *Paper Trade Journal*, May 5, 1927.

LIGHT-COLORED FIRST-COAT ENAMELS FOR SHEET IRON.

IT IS very common practice in the manufacture of vitreous enamel-coated sheet iron and steel articles to apply first a coating which is a dark color, and over this to apply two coats of white enamel to give the desired brilliant white finish. The first coat is dark because it contains cobalt and other oxides which enhance its ability to adhere to the iron, but if a white first coat can be applied which is sufficiently adhesive, a total of two coats will suffice and a saving in manufacturing costs results. There is now in progress at the bureau an investigation to eliminate the necessity for the dark first coat, either through development of a light-colored enamel which by virtue of its composition has an adhesiveness comparable to that of a cobalt enamel, or through improving the technique of application so that a white enamel will adhere sufficiently, or both.

Several different opacifying materials, added in the raw state to the enamel glass before grinding, have been studied to determine their suitability for use in a first coat. These materials included tin oxide, zirconium oxide, zirconium silicate, and sodium antimonate. Of these sodium antimonate and zirconium oxide gave promise of satisfactory behavior for this use.

Since the problem is intimately concerned with adherence of enamels, the development of a test of adherence has been undertaken. Iron plates, one by three inches, are sprayed with enamel and placed in the furnace on a special rack designed so that one of the plates is about one-eighth inch above the other, with an overlap of one inch. When the enamel is matured the pieces are clamped together with special tongs, so designed that the same pressure will be exerted every time. After cooling, the adhering pieces, having a contact area of one square inch, are pulled apart just as if the tensile strength of the iron were being tested. Sometimes the failure occurs in the enamel itself and sometimes in the bond between the enamel and the iron. Although the test has not been completely developed, it shows promise of giving valuable relative data.

STANDARDS YEARBOOK.

In the April number of the *Technical News Bulletin*, mention was made of the new "Standards Yearbook" which was then in press. This publication was released on April 19th, and since that time it has met with a ready sale.[2] Apparently many manufacturers, industrial experts, engineers, purchasing agents, and libraries have found that the book fills a real need.

The free edition was extremely limited and the initial distribution was confined to agencies which coöperated with the bureau in preparing the book, Government departments, and certain standardizing institutions.

Suggestions and criticisms of the "Yearbook," the first one ever issued by the bureau, will be gladly received by the committee having the work in charge. It is the bureau's desire to make the next issue of even greater value to all those interested in standardization.

DIRECTORY OF COMMERCIAL AND COLLEGE LABORATORIES.

The Bureau of Standards makes many tests for the Government, the industries, and the public. Special provision was made for the Government testing in the act creating the bureau. The volume of this work for the Federal Government makes it imprac-

[2] For sale by the Superintendent of Documents; price, $1.

ticable for the bureau to carry out tests for private institutions and individuals if commercial laboratories can do the work. To inform interested persons of the location of other laboratories, the bureau has compiled a directory of the 207 commercial testing laboratories throughout the country, together with indications of the types of commodities which they are prepared to test. Special care has been exercised to make this list complete. It has now been issued in printed form as *Miscellaneous Publication No. 90 of the Bureau of Standards.*[3] Publication was decided upon because of the desirability of making available to all those interested the location of independent testing services throughout the country, and in anticipation of a marked increase in the demand for such service in domestic and export trade.

Accompanying the list of commercial testing laboratories there is presented a list of the laboratories of 143 colleges which are used not only for purposes of instruction, but also to a considerable extent for research work.

It is believed that the existence of this thoroughly classified directory of testing laboratories will have a number of beneficial effects in promoting the use of specifications, not the least important of which will be the inducement offered to the large number of purchasers who have hesitated to buy on specifications in the past because no adequate service for checking the quality of deliveries appeared to exist.

An outline is given of the certification plan, in accordance with which there have already been compiled 48 lists of manufacturers who have expressed their willingness to certify to purchasers that material supplied on orders based on the indicated 48 United States Government Master Specifications complies with the requirements and tests of these specifications.

[3] May be purchased from the Superintendent of Documents; price, fifteen cents.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

SILVER SULPHIDE SENSITIZING BY BATHING.[1]

By S. E. Sheppard and E. P. Wightman.

By using photographic plates thinly coated with dilute emulsion prepared without sensitizer, it has been shown that it is possible to study certain phases of the sensitizing process and to obtain results in agreement with those previously obtained by sensitizing the emulsion in the fluid state. Two methods of treating plates have been employed. It has been found: (1) That with increasing concentration of sensitizer the sensitivity increases to a maximum. This accords with the previous results as stated. (2) That possibly there is an appreciable extension of the spectral sensitivity by sulphide sensitizing. This is not in entire agreement with previous results and will have to be checked by further work. The presence of optical sensitizers is suspected. (3) That with variation of the hydrogen-ion concentration allylthiocarbamide sensitizing solution in a two-stage sensitizing procedure from a pH of 2 to pH of 4, the degree of sensitizing is about equal. While sensitizing at a pH of 5 appears to give greater increase in speed, it may be merely due to a large increase in fog. (4) That soluble bromides decrease the amount of sensitizing. (5) That sensitizing after desensitizing causes the production of a marked amount of fog. The possible causes underlying some of these facts are discussed in some detail, particularly the existence of a maximum in sensitizing which is ascribed to an ionic lattice strain and ionic deformation together with a competition among the sensitive specks for the silver produced photochemically. It is pointed out, however, that the present work cannot allow of a decision as between this concentration speck theory and the halogen acceptor theory of Hickman.

* Communicated by the Director.

[1] Communication No. 299 from the Research Laboratories of the Eastman Kodak Company and published in *Phot. J.*, **67**, 219, 1927; *Sci. Ind. Phot.*, **7M**; 1926.

SILVER-ATTACKING BATHS AND THEIR EFFECT ON LATENT IMAGE CHARACTERISTICS.[2]

By E. R. Bullock.

OF THOSE substances which are capable of converting silver into a salt or salts of silver, a number clearly destroy the low-exposure portion of the latent image, increase its "inertia," and diminish the sensitivity of any unexposed portion of the emulsion film, while others do not clearly destroy any of the latent image, but diminish its gamma, and do not affect sensitivity. Broadly, the first-named class of substances are relatively strong agents, and the second-named class relatively weak, for the oxidation (ionization) of silver. The difference in the kind of action on the latent image is found to vanish under development conditions allowing no greater accessibility of the sub-surface of the silver halide grains to the developer than to the latent image attacking solution, substances of the first class then merely acting faster than those of the second. As regards the difference in the effect on sensitivity, a suggested explanation is based on the greater ability of substances of the first class to attack silver sulphide, which was found by S. E. Sheppard to be the substance of the sensitivity nuclei in gelatin emulsions. The destruction of these sensitivity nuclei may open the way to the destruction of sub-surface latent image and so account for the observed correlation between sensitivity-diminution and definite destruction of latent image (as distinguished from mere lowering of gamma) for ordinary alkaline development.

It is suggested that the absence of a visible effect on development to the optimum threshold in a low-acidity physical developer may be used as a *criterion of destruction of the latent image*, the destructibility of the sub-surface image being inferred from the destruction of the surface image. In an addendum, some experiments by A. Reychler (*C. A.*, 20, 2463), on the partial-destroying action of an oxidizing bath on a latent image for post-fixation physical development, are discussed.

[2] Communication No. 301 from the Research Laboratories of the Eastman Kodak Company and published in *Sci. Ind. Phot.*, 7**M**; 21–4 *et seq.*, 1927.

CLEANING LIQUIDS FOR MOTION PICTURE FILM.[3]

By J. I. Crabtree and H. C. Carlton.

IT IS necessary to clean motion picture film at various stages in its progress from the laboratory to the theatre in order to remove dirt and grease from the base side of negative or positive film, or dirt and grease which accumulates on negative film during printing. Also during projection film rapidly accumulates dirt and oil which must be removed at intervals if a clear picture is to appear on the screen. For cleaning the base side of negative and positive film a mixture of ammonia, water and alcohol is satisfactory. In order to remove dust and finger markings from negative film before printing, it should be wiped gently with silk plush moistened with pure carbon tetrachloride. A large number of possible solvents for this purpose have been investigated, but pure carbon tetrachloride is the only one which at the same time is non-inflammable, cheap, and does not attack either the film base or the image. When film is repeatedly projected it tends to become brittle as a result of loss of moisture from the gelatin coating and film base owing to the excessive heat to which a film is subjected in the projector. The flexibility of the film may be restored by remoistening the film, which can be accomplished by passing through a bath containing a mixture of water and alcohol. A cleaning liquid is also described which at the same time removes oil and restores the flexibility of brittle film. Such a treating liquid consists of a mixture of carbon tetrachloride, water and tertiary butyl alcohol.

THE RELATION BETWEEN THE SPECULAR AND THE DIFFUSE PHOTOGRAPHIC DENSITIES.[4]

By L. Silberstein and C. Tuttle.

THE purpose of the paper is to derive theoretically a formula correlating the so-called specular and diffuse density of a layer of a light-scattering medium, such as a developed photographic film or plate, and to test it experimentally. In accordance with the

[3] Communication No. 306 from the Research Laboratories of the Eastman Kodak Company and published in the *Amer. Cinemat.*, **8**, 9–10, 20–3 *et seq.*, 1927.

[4] Communication No. 297 from the Research Laboratories of the Eastman Kodak Company and published in *J. Opt. Soc.*, **14**, 365–373, 1927.

actual method of measurement of the two kinds of "density," the specular density (D) is defined as the common logarithm of the ratio of the intensity of the incident light to that of the beam transmitted normally to the layer, which will include some fraction (εI_s) of the scattered light, and the diffuse density (D') as the logarithm of the ratio of the intensities of the incident and the total transmitted light including the whole scattered light. The incident rays are in both cases supposed to be collimated, normally to the layer.

It is assumed that of the direct (collimated) light, of intensity I, penetrating to any depth, each silver grain scatters the amount AI and absorbs $\kappa\omega I$, where ω is the average effective area of a grain, κ a numerical factor, and A a coefficient depending on the wave-length and the size of the grain. Also part of the scattered light (I_s) produced within the layer is absorbed by the grains, and as these show no trace of orientation, the amount absorbed by a grain is again expressed by $\kappa\omega I_s$, *i.e.*, with the same coefficient as for the direct light. The effect of an "infinitesimal" stratum with regard to both scattering and absorption is taken to be proportional to the number of grains contained in it. Under these circumstances the finally emerging intensities I and I_s are found at once by solving a pair of simple differential equations. Whence follow densities $D=\text{Log}\,[I_0/(I+\varepsilon I_s)]$ and $D'=\text{Log}\,[I_0/(I+I_s)]$ as functions of the total number (n) of grains per unit area of the layer. Eliminating n, the required relation between the two densities is found to be

$$10^{-D} = \varepsilon \cdot 10^{-D'} + (1-\varepsilon) \cdot 10^{-\beta D'}$$

where $\beta = 1 + \frac{A}{\kappa\omega}$ and the significance of the fraction is as above. Equivalently, in terms of transparencies,

$$T = \varepsilon T' + (1-\varepsilon)\, T'^{\beta}$$

A device for determining the constants ε,β from two observed pairs of corresponding D, D' being given, the formula is tested on the experimental results obtained with five different kinds of emulsion representative of various grain-sizes, and is found to give in each case a close agreement.

NOTES FROM THE U. S. BUREAU OF MINES.*

BURNING TESTS OF COALS IN HOUSE-HEATING BOILERS.

By S. B. Flagg, P. Nicholls, and C. E. Augustine.

A STUDY for publication of some 550 tests of the burning of 184 different coals from all parts of the United States and from foreign countries has been completed by the fuel section of the Pittsburgh station. These burnings were conducted in four different domestic boilers. The average efficiencies and itemized heat losses have been plotted against the coals and their characteristics. The results are interesting and show very good curve uniformity, and indicate what may on the average be expected from a coal whose composition and burning characteristics are known. Although such plottings have the limitations pertaining to all averaged statistical data, yet it is believed that the publication when issued will be of decided interest to boiler manufacturers and heating engineers.

SPECIFIC HEATS OF METALLIC COMPOUNDS.

By O. C. Ralston.

FOR several years the bureau has been receiving requests from the smelting men to do work on the heats of formation and the specific heats of the important compounds that enter the copper and lead smelting furnaces. During the past spring at a meeting of the Smelter Advisory Committee of one of the experiment stations a group of prominent metallurgists again went on record as recommending work on the "physics of the important mineral substances used in smelting." The Pacific Experiment Station has made determinations of the low-temperature specific heats for a number of metal compounds, and is now undertaking more precise determinations of the heats of formation of the common metal oxides and sulphides. The more expensive equipment has now been ordered and a design of the necessary calorimeter prepared. The calorimeter will incorporate all of the most

* Communicated by the Director.

approved features necessary to very precise heat measurement and will be constructed of tantalum which resists all mineral acids but hydrofluoric acid and is scarcely attacked by alkali solutions. It will be of the adiabatic type and will be automatically operated by use of a photo-electric cell for switching heating current on or off.

EXPLOSIBILITY OF DUST FROM UTAH COALS.

By H. P. Greenwald.

THE Bureau of Mines arranged with the operators of four mines in Carbon County, Utah, to ship 5-ton samples of run-of-mine coal to the bureau's experimental mine for tests of the explosibility of the coal dust. Fifty-seven tests were made on these coals. The average proximate analyses of the coal dusts as prepared for the tests are as follows:

Analyses of Coals Tested, Per Cent.

	Designation.			
	A	*B*	*C*	*D*
Moisture	4.6	2.7	2.6	5.7
Volatile matter	42.2	40.9	38.9	41.7
Fixed carbon	46.7	44.5	47.8	47.3
Ash	6.5	11.9	10.7	5.3
Ash plus moisture	11.1	14.6	13.3	11.0
Ratio of volatile to total combustible	47.5	47.9	44.9	46.9

The purpose of the tests was to determine the amount of inert dust or water that must be added to the coal dust being examined to make this non-explosive under various conditions. Whether or not an explosion will travel entirely through a zone of dust sometimes depends on the method of initiating the explosion.

The four coals required 67, 66, 65, and 62 per cent., respectively, of rock dust in a mixture to prevent the continuance through them of a developed explosion in the absence of gas. The total incombustible content of these mixtures was 71, 71, 70, and 66 per cent., respectively, which includes the inherent moisture and ash of the coal. In mixtures of Pittsburgh coal dust, used as the standard in testing, and rock dust 63 per cent. incombustible is required. Three of the Utah coals are among the most explosible tested in the experimental mine. When there

is gas in the air-current, for each per cent. of gas present up to 5 per cent. there must be 7 to 8 per cent. of additional rock dust. Further details will be found in *Technical Paper 386*, recently issued.

THE INTERPRETATION OF CRUDE OIL ANALYSES. (BUREAU OF MINES HEMPEL METHOD.)

By N. A. C. Smith.

DURING the past six years, the Bureau of Mines has made and published several hundred analyses of typical crude oils and has developed a system of interpreting the analyses and of classifying crudes. It is apparently adequate for classifying crude oils into four main groups designated in bureau work as (1) "paraffin base," (2) "intermediate base," (3) "naphene (asphalt) base," and (4) "hybrid base" crudes, and has also been found very convenient in studying the geological relationships between crude oils, and the probable refinery yields of valuable products from them.

Crude petroleum is a very complex substance. An analytical statement of its properties must, therefore, consist of two distinct parts—first, the report of the results of the determinations made by the analyst in the laboratory; second, the proper arrangement, classification and interpretation of these results so that the analysis, as a whole, will indicate the meaning and the relative importance of the essential characteristics of the oil.

This type of analytical statement is useful in judging many raw materials and manufactured products. To say, for example, that a lubricating oil has a Saybolt Universal viscosity of 300 seconds at 100° F. means little in itself. It is only when other characteristics of the oil are studied in conjunction with the viscosity that a true picture of the oil in question can be obtained. To the experienced refiner of lubricating oils a statement giving simply the viscosity, gravity, and color, and the flash and pour points of the oil will often be sufficient, as he can supply from previous experience his own interpretation of the data furnished by the analyst. To the layman, however, the analyst's figures would be unintelligible.

Organic materials, such as crude petroleum, on account of

their great complexity, require especially careful interpretation if the laboratory results are not to be actually misleading. For example, the percentage of "gasoline" in various crude oils is stated in many of the standard text-books on petroleum. Unless one knows, however, just what each particular author means by the term "gasoline" it is impossible to predict the yield of motor fuel from these published analyses.

The method of analysis adopted by the Bureau of Mines for its surveys of typical crude petroleum includes a modified Hempel-type apparatus and accessories, and involves the distillation of a sample of crude oil under carefully controlled conditions that can be duplicated readily, and the subsequent examination of the series of successive fractions thus obtained. In addition, the gravity, color, viscosity, pour point and the sulphur content of the crude itself are determined. For a fuller discussion of the method, and its application as illustrated by typical analyses of various crudes, the reader is referred to Serial No. 2806 of the Bureau of Mines.

THE FRANKLIN INSTITUTE.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting Held Wednesday, June 1, 1927.)

HALL OF THE COMMITTEE,
PHILADELPHIA, June 1, 1927.

MR. M. M. PRICE *in the Chair.*

The following report was presented for final action:

No. 2868: Filmo Camera and Projector.

These devices are the results of inventions by Mr. Albert S. Howell, of Chicago, Ill., and are intended for the use of a portable camera for taking moving pictures and a projecting apparatus suitable for home use.

The construction of the camera provides a spring-driven mechanism which is started and stopped by a push button and which is controlled, as to speed, by means of an automatic speed governor through which uniform time of exposure is assured.

The film used is about half the width of that used in professional machines and the size of picture is much smaller, so that 400 feet of this width film gives as many exposures as 1000 feet of standard film.

The films are developed as positives so they can be used for projection.

The special features characteristic of this camera are:

(1) A spy-glass view finder.

(2) The interchangeability of lenses.

(3) Variations of speeds.

(4) An accurate film indicator which gives both the number of feet of film already used and the number still to be used.

(5) A wide range of focal distance with the universal lens.

(6) A rotary shutter with a 216° opening.

(7) A high grade of workmanship with extreme accuracy in construction, comparing favorably with the high-class professional camera made by this company.

The projector is of simple design and rugged construction. Its especial advantages are:

(1) The quickness with which it can be adjusted.

(2) Its folding frame which enables the picture to be thrown on any direction of screen.

(3) The compactness with which it can be folded for transportation.

(4) The ease with which it can be threaded and operated.

(5) The fact that it can be reversed in direction or brought to a stand-still for the examination of a single picture.

This report recommended the award of the John Price Wetherill Medal to Mr. Albert S. Howell, of Chicago, Ill., "in consideration of the ingenuity shown in successfully embodying the principles of the professional moving picture camera and projector in instruments especially adapted to the use of the amateur and for the excellence of construction of these instruments."

The following report was presented for first reading:

No. 2874: A New Sculpturing Method.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, June 8, 1927.)

RESIDENT.

MR. JAMES J. FITZPATRICK, Chemical Engineer, Delaware Avenue and Tasker Street. For mailing: 2113 Summer Street, Philadelphia, Penna.

DR. HENRY WINSOR, Physician, Haverford, Penna.

CHANGES OF ADDRESS.

COLONEL G. L. ANDERSON, Delafield, Wis.

MR. JOHN ASHHURST, Free Library of Philadelphia, Logan Square, Philadelphia, Penna.

MR. FRANK SHAW CLARK, Stone and Webster, 49 Federal Street, Boston, Mass.

MR. CHARLES H. COLVIN, 8 Willow Place, Brooklyn, N. Y.

MR. J. S. DUNCAN, 4418 Drexel Boulevard, Chicago, Ill.

MR. ARTHUR FALKENAU, "The Lemises," Katanah, N. Y.

MR. FRANCIS W. HARTZEL, 414 Hellerman Street, Philadelphia, Penna.

MR. T. CHALKLEY HATTON, 490 Broadway, Milwaukee, Wis.

MR. F. E. IVES, 1753 North Fifteenth Street, Philadelphia, Penna.

MISS EMILY E. HOWSON, Glen Moore, Chester County, Penna.

MR. W. M. C. KIMBER, 999 East Haines Street, Germantown, Philadelphia, Penna.

DEAN R. B. MOORE, Chemistry Department, Purdue University, Lafayette, Ind.

MR. STEPHEN H. NOYES, Assistant to Chief Engineer, The Pennsylvania Railroad, Philadelphia Improvements, 400 Fox Building, Philadelphia, Penna.

MR. SAMUEL B. SKLAR, Hotel Statler, Euclid Avenue at East Twelfth Street, Cleveland, Ohio.

MR. C. F. SULZNER, 236 S. W. Second Street, Miami, Fla.

MR. CARL D. ULMER, in care of The Koppers Company Laboratories, Mellon Institute, Pittsburgh, Penna.

NECROLOGY.

Mr. F. J. Clamer, Collegeville, Penna.

LIBRARY NOTES.

RECENT ADDITIONS.

BAILEY, C. H.—The Chemistry of Wheat Flour. 1925.

BRAYMER, DANIEL H., and ROE, A. C.—Repair Shop Diagrams and Connecting Tables for Induction Motors. 1927.

BURRELL, GEORGE A.—The Recovery of Gasoline from Natural Gas. 1925.

COMPTON, ARTHUR H.—X-rays and Electrons. 1926.

CORSON, M. G.—Aluminum: the Metal and Its Alloys. 1926.

DEWAR, SIR JAMES.—Collected Papers. Edited by Lady Dewar. Two volumes. 1927.

DUNOÜY, P. LECOMTE.—Surface Equilibria of Biological and Organic Colloids. 1926.

EDDINGTON, A. S.—Stars and Atoms. 1927.

EVERS, NORMAN.—The Chemistry of Drugs. 1926.

GEIGER, H., and SCHEEL, KARL, Ed.—Handbook der Physik. Bände 1, 2, 7, 9, 11, 14, 15, 17, 22–24. Eleven volumes. 1926–1927.

GIFFORD, J. WILLIAM.—Lens Computing by Trigonometrical Trace. 1927.

GREEN, GEORGE ROSS.—World's Columbian Exposition, 1893. Reports on Constructional Work, Power Plant and Boiler House. 1893.

HALE, GEORGE ELLERY.—Collected Papers. Six volumes. 1890–1927.

Handbook of the Central High School of Philadelphia, 1926–1927. 1926.

HEDGES, E. S., and MYERS, J. E.—The Problem of Physico-chemical Periodicity. 1926.

HEVESY, GEORG VON.—Die seltenen Erden vom Standpunkte des Atombaues. 1927.

HOAR, ROGER SHERMAN.—Patents: What a Business Executive Should Know about Patents. 1926.

Jahresbericht über die Leistungen der chemischen Technologie für das Jahr 1926. 72 Jahrgang. Two volumes. 1927.

LODGE, SIR OLIVER.—Science of To-day. 1927.

LOEB, LEONARD B.—Kinetic Theory of Gases. 1927.

LORENTZ, H. A.—Lectures on Theoretical Physics Delivered at the University of Leiden. Authorized translation by L. Silberstein and A. P. H. Trivelli. Volume I. 1927.

LOVE, A. E. H.—A Treatise on the Mathematical Theory of Elasticity. Fourth edition. 1927.

MCKAY, HERBERT C.—Motion Picture Photography for the Amateur. 1924.

MCKEE, RALPH H., and Others.—Shale Oil. 1925.

MORGAN, GEORGE.—The City of Firsts. 1926.

PANNELL, ERNEST V.—High Tension Line Practice: Materials and Methods. 1926.

PONSINET, J.—Principes de l'Électrochimie. 1927.

PRINCE, DAVID CHANDLER, and VOGDES, FRANCIS BROOKE.—Principles of Mercury Arc Rectifiers and Their Circuits. 1927.

RAIZISS, GEORGE W., and GAVRON, JOSEPH L.—Organic Arsenical Compounds. 1923.

SMITH, EDGAR F.—Old Chemistries. 1927.

SPOEHR, H. A.—Photosynthesis. 1926.

SUTERMEISTER, EDWIN.—Casein and Its Industrial Application. 1927.

TAGGART, ARTHUR F.—Handbook of Ore Dressing. 1927.

THOMAS, EDWARD.—The Law of Chemical Patents. 1927.

WALL, E. J.—Photographic Facts and Formulas. 1927.

WELLS, H. GIDEON.—The Chemical Aspects of Immunity. 1925.

WIEN, W., and HARMS, F., Ed.—Handbuch der Experimentalphysik. Band 21. 1927.

WILSON, JOHN ARTHUR.—The Chemistry of Leather Manufacture. 1923.

Zeitschrift für anorganische und allgemeine Chemie. Generalregister der Bände 51–100 (1906–1917). 1922.

BOOK REVIEWS.

THE COLLECTED PAPERS OF SIR JAMES DEWAR. Edited by Lady Dewar, with assistance of J. D. Hamilton Dickson, M.A., H. Munro Ross, B.A., and E. C. Scott Dickson, B.A., Ph.D. Two volumes, xxxi–1489 pages, 8vo, many illustrations, portrait. Cambridge, University Press; New York, The Macmillan Company, 1927.

"Lives of great men all remind us we can make our lives sublime, and departing leave behind us footprints on the sands of time." Thus in the manner of Walt Mason and applying the sentiments of Longfellow, we may express the value of this volume. Its matter, with the exception of two papers, has been before the world for some time for information and criticism. The contributions begin in the late sixties of the last century and terminate a few years back of the present time. The papers are arranged in chronologic order, with a few exceptions, and contain many excellent illustrations. The volumes constitute a vivid picture of the progress of physics and chemistry in their relations for over half a century, for Dewar was a most industrious and original worker. The world owes a great deal to him. Pure science has received large contributions from his labors and applied science has, among many other important contributions, the vacuum bottle.

The list of papers shows the wide range of Dewar's interest in science, and beginners in research work in chemistry and physics will find a great deal to guide and suggest in the perusal of these volumes. It is obviously not appropriate to review individual articles for the great mass of the material has, as is noted above, long since passed into the literature of the sciences concerned and received its approval and acceptance.

In the first paper in the first volume suggestions are given for models of structural formula. These are interesting as the early efforts in this direction which has of recent years become so striking a feature of chemistry. Various complicated forms consisting of hinged pieces of wood, mostly somewhat fantastic, are given. The date of this paper is 1867. About a year later Dewar brought before the British Association for the Advancement of Science a model devised by Kekulé to illustrate the structure of organic compounds. This was a sphere with four equal wire arms joining in the centre of the

sphere with the angles of an imaginary equilateral tetrahedron. He demonstrated to the meeting how this model could be used for illustrating the structure of both fatty and aromatic compounds. As is now well known, the model has been simplified into the regular tetrahedron and largely used in the illustration of the structure of carbon compounds. HENRY LEFFMANN.

INTRODUCTION TO THE HISTORY OF SCIENCE, Volume I, from Homer to Omar Khayyam. By George Sarton, Associate in the History of Science, Carnegie Institution of Washington. xi–839 pages, large 8vo. Baltimore, The Williams and Wilkins Company. Price, $10.

It is very difficult to value this book. That it is a work of great erudition in which the extensive array of facts covered has been well ordered and clearly set forth is evident, but the size of the volume is somewhat deterrent. One is tempted to speak as Macaulay did in reviewing the Life of Lord Burghley by Nares in which he gave the cubic contents and the weight of the volume, but the size of Nares' book was due largely to his prolixity for, as Macaulay said, his title-page was as long as a paragraph and his paragraphs were as long as chapters. The size of this book is due to the carefully selected and carefully expressed information which it furnishes. It is the first of a series of volumes. How many will follow does not seem to be indicated. Possibly the author has not yet determined how vast will be his treatment of the subject. It begins as one might expect, in the early period of Greek literature. The poems ascribed to Homer are probably composite in nature, collected by editors and more or less extended by later writers. At the outset we may remark that the great poem is very generally misunderstood, being designated a history of the Trojan War, whereas it is only a description of a brief period during the wrath of Achilles when he sulked in his tent from which he was aroused by the death of his friend Patroclus. The Ten Years' War which figures so largely in later history is probably a mere invention. The whole affair may have been merely a summer foray. At any rate the philosophic historian will find in the account in the Iliad, taken into consideration with the geographic and physiographic conditions of the district, that the war was essentially economic in its character, the determination of the Greek states to break the control of the commerce between Asia and Europe which the Trojan cities were exercising. It has been shown pretty clearly that both the land and water transportation could be easily controlled from Troy. It is, of course, not impossible that some sentimental or exciting incident may have set fire to the slumbering conditions, for instance, like the amputation of Jenkin's ear, the suggestion of the king of Prussia to put a Hohenzollern on the throne of Spain, the sinking of the *Maine* or the assassination of the Archduke and his wife at Sarajevo, that like all these the abduction of Helen might have stirred the Greeks to action and brought about a temporary union of those unfortunately unfriendly states. Not much of value to science can be gained from Homer. The poems are more satisfactory as pictures of the social standards and the beliefs of the age. The Odyssey mentions the hardening of steel by quenching in water. We note that in Homer's day the dawn was red, for he speaks of the "rosy-mantled dawn." It was said indeed some years

ago by a writer on color problems that in those days there may have been a sort of color blindness, for while the Iliad is profuse in its use of adjectives, the blue sky and the green grass are never mentioned.

Coming to a later date we encounter the real Greek scientists, the men who studied nature to elucidate the truth without reference to purpose or particular origin. About the fifth century before our present era, the atomic theory was put forth with ideas of the structure of matter almost identical with those that were held up to the beginning of this century. These philosophers coined from their own language the adjective "atomos" (indivisible), and so was born our word "atom," familiar throughout the whole of physical and chemical science but much altered in its basic significance.

It is interesting to compare the attitude of the Greek with that of the Hebrew. Our present civilization is very largely the product of the mingling of those streams. Our moral standards and ethical concepts are Jewish. We are guided, at least some of us are, by the Ten Commandments. The Hebrews, however, did not trouble themselves about the nature of phenomena but rather the significance thereof. The Greeks were intent on the study of phenomena in their nature. This is well shown in a comparison between the attitude of the two races concerning the rainbow. The Hebrew word means the "weapon" and its appearance in the heaven was to indicate that the Deity had hung up his arms and had ceased to make war on mankind. Not so the Greek. He had no notion of a purpose, but to him the question was "How is this phenomenon caused?" A suggestion was made by a Greek philosopher that it was due to the action of the light upon raindrops, but he did not know the composite nature of white light and therefore could not account for the colors.

In this book are collected data concerning practically all of the philosophers of the ancient world between the periods indicated in the title. That much of this is of very little value in the study of science must be admitted, and it perhaps would have been better to have printed this material in small type, leaving the more important data to be read by those who are not pursuing the subject into its very depth. However, the intention of the volume is probably to place on record all material that is obtainable, making it rather a work of reference than a work to be read or a text-book. An interesting picture of the change that has taken place in the world's course is presented on pages 10 and 11, where the decline of civilization under Roman influences is presented. The Romans copied the Greek models in art, literature, science, less in politics, but they did not improve them. Militarism and the passion for conquest became the dominant impulses and the moral and ethical conditions passed into collapse with the enormous increase of wealth and power that accumulated in the great cities of the empire. Macaulay has very aptly expressed in one of his poems the contrast between Rome and the two great Mediterranean powers of its day. In the speech put into the mouth of a seer who is speaking at the dedication of Rome, the future age is pictured:

> "Leave to the sons of Carthage the rudder and the oar,
> Leave to the Greek his marble nymphs and scrolls of worldly lore,
> Thine, Roman, is the pilum, Roman, the sword is thine,
> The even trench, the glittering mound, the legion's ordered line."

Into this decaying civilization in the fifteenth year of the reign of Tiberius Cæsar when Herod Antipas was ruler of Galilee and Pontius Pilate was procurator of Judea, an orthodox Jew, Yehoshua bar Yusef—Joshua, son of Joseph—began his ministry, preaching a reform Judaism. After less than a year his personal relations to the movement were terminated by his execution for sedition, but he had planted a fertile seed among the Jewish people and it extended very soon into the Greek world. In a few years the Greek powers of the Christian Church became the dominant portion of the organization and our records of this ministry are in Greek and most of its earliest history. The author of this book very frankly states that as a result of this triumph of a moral and ethical reform which Christianity brought about there was a loss of scientific interest. This was naturally the case because the Christians had at their command a Book, an extensive record of history and an ample formulation of morals. This Book contained an account of the creation of the world and much information concerning the nature and relations of the living creatures in it. Why go further? So we find that from the time that Christianity triumphed in its control of the affairs of the Roman Empire through a long period until certain conditions awakened men to a broader conception, science was partly in abeyance. Not wholly, for the human mind is filled with curiosity and even in the darkest ages there are points of brilliancy that deserve our study and commendation. The story of this development will be found set forth in great detail and vivid language in the work before us which is commended strongly to the attention of those who wish to know the full facts of the early history of science. HENRY LEFFMANN.

STATISTICAL MECHANICS WITH APPLICATIONS TO PHYSICS AND CHEMISTRY. By Richard C. Tolman, Ph.D., Professor of Physical Chemistry and Mathematical Physics, California Institute of Technology. 334 pages, 22 × 15 cm. cloth. New York, The Chemical Catalog Company, Inc. Price, $7.

The methods of actuarial science in which statistics bearing upon the duration of life are adjusted and interpreted by the application of mathematical processes based upon the laws of probability have long since proved eminently satisfactory, despite the myriad complex and unrecorded phenomena that contribute to the final result. Refinements of recent times with the progress of evolution in the physical sciences have brought to light phenomena to which established laws of an earlier period have not proved applicable. The old structure which many hoped would eventually lead to a definitely deductive system proved to be an expedient rather than a finality. A better expedient was needed, and investigators in physical science have not been tardy in recognizing the value of the calculus of probabilities as an immediately applicable solution to their problems.

The important contributions to physical science which have accrued from the adoption of this method in recent times have aroused widespread interest among investigators in molecular physics and physical chemistry. The details of the application of these statistical methods are more or less scattered among these papers and the author's object has been to gather between two covers

a deductive account of this statistical mechanics which has already yielded rich returns, with the addition of the results of his own researches. The work is in four main divisions: (1) The classical statistical mechanics; (2) Introduction of the quantum theory; (3) Application to molecular processes; (4) Rate of physical change; and a concluding chapter on the relation between statistical mechanics and thermodynamics. The mathematician's treatment of deriving specific applications from general formulas is employed and the reader should be familiar with the complexities of the phenomena of physical science and adept in the mathematical processes of physical investigation. The work greatly lightens the task of advanced students of physics and physical chemistry by presenting a logical account and the application of the statistical method in a single volume. LUCIEN E. PICOLET.

EXPERIMENTAL STUDIES ON THE EFFECT OF ETHYL GASOLINE AND ITS COMBUSTION PRODUCTS. Report of the United States Bureau of Mines to the General Motors Research Corporation and the Ethyl Gasoline Corporation. By R. R. Sayers, A. C. Fieldner, W. P. Yant and B. G. H. Thomas. xviii–447 pages, illustrations and plates, 8vo. Washington, U. S. Department of Commerce, 1927.

This is the detailed account of the investigations made by the U. S. Bureau of Mines in accordance with agreements with the General Motors Company and Ethyl Gasoline Corporation. The investigations were made in consequence of the introduction of tetrethyl lead as an "anti-knock." The subject attracted a great deal of attention a few years ago in consequence of numerous fatal cases among those engaged in the manufacture of the lead compound. "Ethyl gasoline" appears to be a sort of euphemism for fuel that contains tetrethyl lead. The investigation recorded in the report was conducted by Messrs. Sayers, Fieldner, Yant and Thomas. It was very comprehensive and thorough. All the information obtained is set forth in the report together with the methods employed. HENRY LEFFMANN.

QUELQUES IDÉES SUR L'ÉLECTRODYNAMIQUE. Théories nouvelles sur l'oscillateur de Planck et le mouvement autonome exposées devant la Société française de Physique. Par R. Ferrier, Ingénieur des Ponts et Chaussées; préface de Paul Painlevé, Membre de l'Institut. 48 pages, 22 × 13 cm., paper. Paris, Albert Blanchard, 1927. Price, 5 francs.

Extract from the Preface: The theory of the electron is to-day still the least perfect part of electrodynamics. The author modifies it profoundly in order to escape the difficulties which it introduces. The objection as to the stability of the Bohr atom which he raises in the case where the initial energy is less than the final energy is a serious one. Otherwise, in effect, the energy necessary for a stabilization of the movement must be furnished by extraneous electric systems—and thus the Bohr atom does not absolutely merit the name atom, because it cannot be conceived separately—or else this energy is supplied from the vacuum itself, and then this vacuum must be considered substantial, and we return to the idea of a material ether.

It is upon that part that M. Ferrier dwells. Will he encounter the diffi-

culties of others before him who have attempted to construct a theory of the ether? To avoid them, he constructs a system of dynamics which in ordinary applications should agree with classical dynamics but of which the principles are daring, even paradoxical at first view. The author admits of no special origin of reference. He considers only the form of a figure composed of points and the variations as a function of time of that form; that is, of the mutual distances of those points. At each instant these variations by hypothesis are governed by the purely internal occurrences within the system, or with respect only to its configuration. These autonomous movements are thus stable, at least so the author assumes, because they proceed from a condition of maximum of an integral.

The remainder of the structure is sketchy and its completion is not at all easy. However bold may appear these principles of the preceding theory—but is there any electrodynamic system which escapes this criticism?—it would be interesting by reason of their very daring to develop their consequences up to controllable conditions. LUCIEN E. PICOLET.

PROBLEMS OF MODERN PHYSICS. A course of lectures delivered in the California Institute of Technology. By H. A. Lorentz, Professor in the University of Leiden, edited by H. Bateman, Professor in the California Institute of Technology. vi–312 pages, 20 × 14 cm., cloth. Boston, Ginn and Company. Price, $3.60.

The upheavals which have taken place of recent years in the fundamentals of physical science have scarcely yet attained the stable state of universal acceptance, much less have these profound changes, in so far as they have been accepted, assumed such didactic form that the student of to-day may have nothing to unlearn later. The new concepts of space and time relationship, of atomic structure and radiation, and the statistical method of deriving physical laws, in the hands of brilliant investigators in many lands, have stimulated advances to leaps and bounds, and a formative period rather than one that permits codification is at hand.

It is fortunate for students here who would maintain contact with these revolutionary changes that a great leader like Professor Lorentz has been inclined to visit this country and discuss these developments at length. His lectures at the California Institute of Technology in 1922 are now recorded in this volume with an appendix containing later amplifications of parts of his discourse. Under the general caption of "Light and Matter with some considerations on Relativity" the entire field of fundamental physics is covered. A few of the many separate topics are here mentioned: The propagation of light; The electromagnetic equations; The principle of Huygens; Flow of energy in a beam of light; Solutions of Maxwell's equations for the ether; The field equations of the electron theory; Difficulties in the theory of radiation; The Michelson-Morley experiment; Relativity and the principles of the conservation of energy; Interference and the quantum theory.

Highly mathematical as necessarily is the discussion, the reader's task is much lightened by the liberal verbal explanations within which the mathe-

matical deductions are interwoven. It is a timely presentation of a profound subject from the fountain-head of its most active development, an excellent piece of editorial work and a credit to its publishers.

LUCIEN E. PICOLET.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Report No. 257, Pressure Distribution over a Wing and Tail Rib of a VE-7 and of a TS Airplane in Flight. By J. W. Crowley, Jr. 37 pages, illustrations, quarto. Washington, Government Printing Office, 1927. Price, fifteen cents.

This investigation was made at the Langley Memorial Aeronautical Laboratory of the Committee to determine the pressure distribution over a rib of the wing and over a rib of the horizontal tail surface of an airplane in flight and to obtain information as to the time correlation of the loads occurring on these ribs. Two airplanes, VE-7 and TS, were selected in order to obtain the information for a thin and a thick wing-section. In each case the pressure distribution was recorded for the full range of angle of attack in level flight and throughout violent manœuvres. Particular attention was given to the high and low angle of attack conditions. The results show: (*a*) That the present rib load specifications in use by the Army Air Corps and the Bureau of Aeronautics, Navy Department, are in fair agreement with the loads actually occurring in flight, but could be slightly improved; (*b*) that there appears to be no definite sequence in which wing and tail surface ribs reach their respective maximum loads in different manœuvres; (*c*) that in accelerated flight, at air speeds less than or equal to 60 per cent. of the maximum speed, the accelerations measured agree very closely with the theoretically possible maximum accelerations. In manœuvres at higher air speeds the observed accelerations were smaller than those theoretically possible.

Report No. 259, Characteristics of Propeller Sections Tested in the Variable Density Wind-tunnel. By Eastman N. Jacobs. 16 pages, illustrations, quarto. Washington, Government Printing Office, 1927. Price, ten cents.

Tests were carried out in the variable density wind-tunnel of the Committee on six airfoil sections used by the Bureau of Aeronautics as propeller sections. The sections were tested at pressures of 1 and 20 atmospheres corresponding to Reynolds' numbers of about 170,000 and 3,500,000. The results obtained, besides providing data for the design of propellers, should be of special interest because of the opportunity afforded for the study of scale effect on a family of airfoil sections having different thickness ratios.

Report No. 260, The Effect of a Flap and Ailerons on the N.A.C.A.-M6 Airfoil Section. By George J. Higgins and Eastman N. Jacobs. 18 pages, illustrations, quarto. Washington, Government Printing Office, 1927. Price, ten cents.

Here are given the results obtained at the Langley Memorial Aeronautical Laboratory on an N.A.C.A.-M6 airfoil, fitted with a flap and ailerons, and tested in the variable density wind-tunnel at a density of 20 atmospheres. Airfoil characteristics are given for the model up to 48° angle of attack with the flap set at various angles, and also with the ailerons set at similar angles. The approximate lift distribution and the centre of pressure variation along the span

are determined with the model at 18° angle of attack and with the ailerons displaced 20°. Approximate rolling moment and yawing moment coefficients are determined for the various aileron settings.

A comparison of the calculated angles of zero lift and the calculated lift and moment coefficients with those observed is given in the appendix.

Report No. 265, A Full-scale Investigation of Ground Effect. By Elliott G. Reid. 7 pages, illustrations, quarto. Washington, Government Printing Office, 1927. Price, five cents.

This report describes flight tests which were made with a Vought VE-7 airplane at the Langley Memorial Aeronautical Laboratory, to determine the effects of flying close to the ground.

It is found that the drag of an airplane is materially reduced upon approaching the ground and that the reduction may be satisfactorily calculated according to theoretical formulas.

Several aspects of ground effect which have had much discussion are explained.

PUBLICATIONS RECEIVED.

Introduction to the History of Science. Volume 1, from Homer to Omar Khayyam, by George Sarton, Associate in the History of Science, Carnegie Institution of Washington. 839 pages, quarto. Washington, Carnegie Institution of Washington, 1927. Price, $10.

The Structure of the Atom, by E. N. daC. Andrade, D.Sc. Third edition, revised and enlarged. 750 pages, illustrations, plates, 8vo. New York, Harcourt, Brace and Company, 1927.

Traité de Balistique Extérieure, par l'Ingenieur général P. Charbonnier, Tome II, Balistique Extérieure Rationelle. Les Théories Balistiques. 797 pages, illustrations, 8vo. Paris, Gaston Doin et Cie., Gauthier-Villars et Cie., 1927. Price, 150 francs.

The Principles of Petrology. An introduction to the science of rocks by G. W. Tyrrell. 349 pages, illustrations, 12mo. New York, E. P. Dutton and Company, 1927. Price, $3.25.

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Statistical Mechanics with Applications to Physics and Chemistry, by Richard C. Tolman, Ph.D. American Chemical Society Monograph Series. 334 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1927. Price, $7.

The General Theory of Thermodynamics. An introduction to the study of thermodynamics by J. E. Trevor, Ph.D. 104 pages, illustrations, 12mo. Ginn and Company, 1927. Price, $1.60.

Quelques Idées sur l'Électrodynamique. Théories nouvelles sur l'oscillateur de Planck et le mouvement autonome exposées devant la Société française de Physique, par R. Ferrier, avec une preface de Paul Painlevé. 48 pages, 12mo. Paris, Librairie Scientifique Albert Blanchard, 1927. Price, five francs.

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Stefan Jellinek. 101 pages, illustrations, colored plate, 8vo. Leipzig, Franz Deuticke, 1927. Price, in paper, 12 marks; bound, 14 marks.

Insulating Oil. A list of references (1900–1925) in the New York Public Library, compiled by Arthur W. Fyfe, Jr., Science and Technology Division, with an introduction by J. L. R. Hayden, Research Engineer, General Electric Company. 71 pages, 8vo. New York, The New York Public Library, 1927. Price, fifty cents.

Ventilation of Vehicular Tunnels. Report of U. S. Bureau of Mines to New York State Bridge and Tunnel Commission and New Jersey Interstate Bridge and Tunnel Commission by A. C. Fieldner, Yandell Henderson, J. W. Paul, R. R. Sayers and others. 171 pages, illustrations, 8vo. Reprinted from *Journal of American Society of Heating and Ventilating Engineers,* January-December, 1926. New York, February, 1927.

Experimental Studies on the Effect of Ethyl Gasoline and Its Combustion Products. Report of the U. S. Bureau of Mines to the General Motors Research Corporation and the Ethyl Gasoline Corporation by R. R. Sayers, A. C. Fieldner, W. P. Yant, and B. G. H. Thomas. 447 pages, illustrations, plates, 8vo. Washington, Bureau of Mines, 1927.

Link-Belt Company, Book 575. "Handling Things—from Where They Are to Where You Want Them." 96 pages, illustrations, 8vo. Chicago, Link-Belt Company, 1927.

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CURRENT TOPICS.

Annual Report of the Director of the Department of Terrestrial Magnetism, Carnegie Institution of Washington, for the Year 1925–1926. LOUIS A. BAUER, Director.—When the riddle of the earth's magnetism is read and its explanation becomes a commonplace of text-books on physics, much of the credit for the achievement will, no doubt, be due to the activities set forth in this and in other annual reports of the Department of Terrestrial Magnetism.

The wide range of the phenomena concerned can be seen from this, "While, on the average, there is a very high correlation from year to year during the eleven-year solar cycle between sun-spottedness and the earth's magnetic disturbances, the correlation does not seem to be indicative of immediate cause and corresponding effect, but rather that sun-spots, solar prominences, etc., and magnetic storms are all effects of one, as yet undiscovered, cause which may simultaneously affect the condition of the entire sun. However, there is another type of disturbance shown by fluctuations in the earth's magnetism, earth-currents, atmospheric electricity, and polar lights, revealing a double periodicity in the course of the year, which has not yet been satisfactorily explained by changing sun-spottedness or changing efficiency of a given sun-spot area during the year. We are dealing here with effects resulting from the annual motion of the earth around the sun. It may turn out that because of the fact that the earth acts like a great magnet and behaves like a great electroscope, we have brought to us daily on the strips of paper wound around the recording drums of magnetographs and of electrographs, photographic evidences of solar and cosmical changes which other means fail to reveal to us in their entirety. And judging from reports received during the present period of marked increased solar activity, radio-reception measurements may effectively supplement the magnetic and the electrical data."

On the other hand, a coöperative investigation conducted by the Geophysical Laboratory and the Department of Terrestrial Magnetism led to results that have a distinctly mundane bearing. The critical temperature of magnetization of nickel, iron and magnetite was found to be lowered a little by the application of pressure. When these substances are heated it is at the critical temperature that they cease to be magnetic. "Accordingly the argument cannot be successfully combated that at a depth of 10 or 12 miles below the earth's surface the temperature of magnetizable materials is such as to exclude the possibility that any portion of the earth's magnetism can arise from materials at that depth."

The Department has continued its work in atmospheric electricity

by extending the field of observations, by developing instruments and by securing the coöperation of individuals and organizations. It is not difficult to understand why the Radio Corporation of America takes an interest in such work, but it is not quite so clear why the Wistar Institute of Anatomy and Biology should make continuous records of potential gradient. However, the activities of the Department turn out to be of value in other unexpected places. The Cerro de Pasco Copper Corporation received meteorological data from the observatory maintained in Peru in order that it might use them in studying the connection between a disease of cattle and rainfall.

The non-magnetic vessel, *Carnegie,* was not in commission during the period covered by the report. Plans were made, however, for future voyages on which much biological and oceanographic work, as well as magnetic and electrical observations, will be carried on. The annual record of operations on land never fails to contain a romantic element. In Africa observations were made in Morocco, the Canaries, in Nigeria, and on the Gold Coast at Accra, where it is, according to Robert Hamill Nassau, that, quite literally, "Afric's sunny fountains roll down their golden sands." On Amundsen's vessel, *Maud,* in winter quarters off Siberia, absolute magnetic observations and records of declination and of potential gradient were secured for at least six months. The regular observatory work was continued at Watheroo in Australia, Huancayo in Peru, Apia in Samoa, as well as in the Washington laboratory. G. F. S.

Latest Annual Values of the Magnetic Elements at Observatories. Compiled by J. A. FLEMING. (*Terrestrial Mag. and Atmos. Elect.,* March, 1927.)—No less than sixty observatories are listed here, some with familiar names, such as Kew, Greenwich, Potsdam, Prague and Valencia, and others with unfamiliar designations—Swider, Matochkin Shar, Alibag and O'Gyalla. The last is in Hungary, not in Ireland. In addition to the continental observatories there are those on islands, great and small, Greenwich, Uccle, Batavia, Christchurch, Apia, Mauritius and Tananarive. Only eleven of the sixty are located in the southern hemisphere of the earth, and yet this is as large as its northern counterpart. Sitka has the distinction of possessing the largest declination of all, 30° 27′ in 1925, that is, there the north-seeking end of the compass needle pointed 30° 27′ to the east of the geographical north. At Meanook in Alberta the next largest declination exists, 27° 10′, but to the west, though the observatory is not relatively far from Sitka. At Hongkong, Toungoo, Batavia, Antipolo and Alibag the declination is less than one degree.

The inclination, the angle made by a dipping needle with a horizontal line, is largest at Matochkin Shar, on some island north of European Russia, where its value is 80°. Though Meanook is 16° nearer the equator of the earth than the observatory just named, its declination is almost as large, being 74°. This is due to the nearness

of Meanook to the magnetic pole of the earth. In both observatories it is the north-seeking pole that comes to rest below a horizontal plane passing through the centre of the dipping needle. In all the observatories of the southern hemisphere, on the contrary, the south-seeking pole is the one below this plane. Christchurch, New Zealand, has a south inclination of 68°. At Meanook the vertical intensity of the earth's magnetic field is greater than at any other observatory, amounting to 599 c.g.s. units. This again is associated with the fact of its proximity to the magnetic pole of the earth, the direction of the total magnetic force being near to the vertical, and the horizontal component attaining a value of only .13 unit. For observatories near to the magnetic equator, *i.e.*, having a small inclination, the vertical component has, of course, small values. Huancayo, Peru, inclination, 1° N.; vertical component, .005. Kodaikanal, 4° 40′ N., .03. The minimum strength of the horizontal component is found at Matochkin Shar, .0952. G. F. S.

The Dependence of Surface Tension upon Electrical Charge. O. RUFF, G. NIESE and F. THOMAS. (*Ann. d. Phys.*, No. 5, 1927.)—The question of the connection between these two quantities is by no means a new one. Barnett and Merritt at the beginning of the present century made careful experiments and concluded that the tension of a surface charged to 30,000 volts was not thereby changed by an amount exceeding the experimental errors of the observation. The authors develop from theoretical considerations an equation by which the reduction of the surface tension of a drop can be calculated from a knowledge of quantities that can be observed or calculated. They arrive at the effective surface tension by getting the weights of droplets of mercury falling under conditions selected to avoid sources of error. Varying voltages were applied to the drops as they formed and a distinct change of weight and of size was discovered. Thus with no charge the average radius was .131 cm.; with 19,800 volts applied it was .125 cm. and with 31,310 volts .119 cm. In the two cases the surface tensions were respectively 95.4 per cent. and 93.6 per cent. of the surface tension in the absence of charge. These results agree well with the theory previously developed. They find, however, that their theory cannot be applied to the case of drops of water on account of the difficulty of determining the charge on the drops from the amount of applied voltage. G. F. S.

Experiments on the Deflection of Molecular Beams in a Magnetic Field. ALFRED LEU. (*Zeit. f. Physik.*, **41**, Nos. 8 and 9.)—These experiments form a continuation of those of Gerlach and Stern. By an electric heater elements are volatilized in a vacuum and by means of appropriate slits a beam of the issuing vapor is segregated. It passes through a small glass tube where it traverses a non-homogeneous magnetic field, and impinges on a surface cooled by liquid air. The image there produced is observed by a microscope

through a window in the apparatus. During the experiment the pressure within was kept at something less than 10^{-5} mm. mercury. Zinc, cadmium, potassium, sodium and thallium were tested. With zinc and cadmium the image due to condensation on the cooled surface was just the same in breadth with the magnetic field as without it. In the cases of potassium and sodium, however, the application of the field caused the single deposited line to turn into two parallel lines distinctly separated. With thallium the lines were separated to a much smaller extent. The doubling of the line is caused by the non-homogeneous field deflecting the molecules in the beam, some to one side, the remainder to the opposite side, for those elements possessing molecules with magnetic moments. The molecules of zinc and cadmium are without magnetic moment, while those of the three other elements have moments. Calculation showed that the magnetic moment of sodium and potassium molecules is one Bohr magneton while the corresponding moment for thallium is only one-third as great. This accounts for the smaller separation of the lines in the case of thallium. G. F. S.

The Wave-length of the Green Auroral Line. G. CARIO. (*Zeit. f. Physik.*, **42**, No. 1.)—In recent years this line has been the subject of numerous investigations. Vegard maintained that it was due to nitrogen, while McLennan and Shrum attributed it to oxygen. Babcock in 1923 published in the *Astrophysical Journal* a careful determination of its wave-length, finding it to be 5577.350 ± .005 Ångström units. Now in Goettingen the author measured with great care the wave-length of the green line emitted by oxygen. His value is 5577.348 ± .005 A.U. This agrees so closely with Babcock's result for the auroral line that the identity of the two may be regarded as established and it seems safe to go on to an examination of the consequences of this identification upon the constitution of the upper atmosphere where the aurora originates.
G. F. S.

Salt Consumption of Sheep.—The consumption of sodium chloride by sheep is an important factor in animal husbandry, but has hitherto been studied to but a limited extent. JOHN M. EVVARD, L. C. BROWN, C. C. CULBERTSON, and W. E. HAMMOND (*Agric. Exp. Sta., Iowa State College of Agriculture and Mechanic Arts, Research Bull. No. 94*, 129–173, 1926) find that optimum nutritional results are obtained when salt is given to sheep and lambs in block or flake, and they are permitted to partake of it as they desire. This procedure is preferable to addition of sodium chloride to the ration. The daily salt consumption of a lamb was found to be between 0.001 and 0.019 pound, with an average value of 0.011 pound. The salt requirement and consumption of a fattening lamb varied with the character and composition of the ration. In order

to produce a gain in weight of 100 pounds, the salt consumption ranged between 0.21 and 11.18 pounds, with an average value of 3.78 pounds. The results reported are based on the records of the consumption of sodium chloride by 1306 winter-fed lambs.

J. S. H.

Windows and the Ultra-violet Rays.—The Council on Physical Therapy of the American Medical Association (*Jour. Am. Med. Asso.*, 1927, **88**, 1562–1567) has investigated the transmission of ultra-violet rays through certain window glasses and certain window glass substitutes. Special attention was paid to those ultra-violet rays which play a part in the prevention of rickets. Both physical and biological methods were used in the measurement of the transmission of these rays. The special window glasses, which transmit the ultra-violet rays, are manufactured in two forms which are, respectively, transparent and translucent to the visible rays. The window glass substitutes, which transmit the ultra-violet rays, are composed of a fabric of wire or cloth filled with either a paraffin-like or a celluloid-like substance. The research yielded results which encourage the belief that materials for glazing windows are now available, which do not exclude the health-giving rays of sunlight, while ordinary window glass excludes these rays. J. S. H.

Melting-point of Monopotassium Phosphate.—ALBERT R. MERZ, of the U. S. Bureau of Soils (*Jour. Am. Chem. Soc.*, 1927, **49**, 1511–1512), has prepared pure monopotassium phosphate. It did not melt in contact with its boiling saturated aqueous solution. Heated in a melting-point capillary, it had no true melting-point, but fused with decomposition and evolution of water vapor. Fusion began at 252.6° and was complete at 254.3° C. corrected.

J. S. H.

New Cuprammonium Thiosulphate.—DAVID WILBUR HORN and RULIFF EDSON CRAWFORD (*Am. J. Pharm.*, 1927, **99**, 274–279) have prepared a new complex cuprammonium thiosulphate by the reaction between a concentrated solution of sodium thiosulphate in concentrated ammonia water and a saturated aqueous solution of cupric chloride. The salt deposits as deep blue crystals which are insoluble in water, concentrated ammonia water, and a cold aqueous solution of sodium thiosulphate. They have the formula $Cu_{10}S_{15}O_{18}.9NH_3$. J. S. H.

Iodization of Public Water Supplies.—The theory that endemic goitre is due primarily to a relative or absolute deficiency of iodine in the food is now widely accepted. The prevention of this disease requires minute dosage of iodine, using a palatable, efficient, and harmless preparation at a low cost. One method of conveying and administering iodine is the iodization of public water supplies.

A comprehensive study of this method has been made by ROBERT OLESEN, of the U. S. Public Health Service (*Pub. Health Rep.*, 1927, **42**, 1355–1367).

The physiological requirements of man are satisfied by 0.01 milligram of iodine daily. When the water supply contains 5 or more parts of iodine, calculated as sodium iodide, per 1,000,000,000, the iodine supply of a region is ample. When the iodine supply is deficient, the deficiency may be overcome by addition of minute amounts of sodium iodide to the water supply. This salt readily dissolves in the water. The addition may be made daily, or less frequently. The dosage, naturally, is governed by its frequency. At the present time, iodization of the drinking water is practised in Rochester, N. Y., and Anaconda, Mont., and in Ilkeston and Heanor, England. It has been practised in Sault Ste. Marie, Mich., and Virginia, Minn. From his study of the subject, Olesen concludes that, while iodization cannot be recommended for wider use until stronger evidence concerning its value is forthcoming, nevertheless, iodized water should not be condemned as worthless. J. S. H.

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